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AUTHOR Callahan, Leroy G.
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ABSTRACT

This study examined the school arithmetic development of three groups of students from an urban school district who differed in performance on precisely described number tasks on entry into first grade. The investigation followed pupils' arithmetic development to the completion of third grade, for those normally promoted each year. The two major questions examined were: (1) Are there quantitative and/or qualitative differences in the school arithmetic development of students with differing number abilities on entry to first grade as they proceed through the primary grades? and (2) Are there differences in the school arithmetic programs provided for students with differing number abilities on entry into first grade? Teachers' perceptions of pupil arithmetic, intellectual, socio-emotional, and psycho-motor maturity were found to correspond closely with student number ability on entering first grade across the three years of investigation. Student performance on simple number tasks on first-grade entry appeared quite a valid predictor of pupil performance two and three years later. Arithmetic development appeared closely related to general psychological development. It is suggested that dramatic program adaptations should be considered for students who enter first grade with immature arithmetic development. (MP)

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ED225843

FINAL REPORT

SCHOOL ARITHMETIC DEVELOPMENT: MOVEMENT TOWARD CONCEPTUAL MATURITY OF STUDENTS WITH DIFFERING NUMBER ABILITIES ON ENTERING FIRST GRADE

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LEROY G. CALLAHAN
PRINCIPAL INVESTIGATOR

NIE-G-80-0097

STATE UNIVERSITY OF NEW YORK AT BUFFALO

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Introduction and Overview of the Study

Recent research has examined the development of number skills, such as counting, in young children (Gelman & Gallistel, 1979; Fuson, Richards & Briars, 1982; Steffe, von Glasersfeld, Richards & Cobb, 1982). These intensive efforts have generated some compelling evidence regarding the cognitive contributions of those developing skills to the subsequent development of number concepts. Earlier surveys of children's number abilities on entering kindergarten or first grade (Brownell, 1941; Williams, 1965; Rea & Reys, 1970, 1971), have confirmed that children enter school with a broad range of number skills and abilities. If skills such as counting are essential to subsequent number concept development, and if students enter school with great variations in such skills, it would be of interest to ascertain the school arithmetic development of students entering school with different number abilities. Few, if any, studies have attempted to follow students with differing number abilities through their early school years in order to study subsequent school arithmetic development.

This research examined the school arithmetic development of three groups of students from an urban school district who differed in performance on precisely described number tasks on entry into first grade. The longitudinal study followed the students' arithmetic development to the completion of third grade, for those normally promoted each year. The study addressed two major questions.

1. Are there quantitative and/or qualitative differences in the school arithmetic development of students with differing number abilities on entry to first grade as they proceed through the primary grades? If so, what is the nature of these differences?
2. Are there differences in the school arithmetic programs provided for students with differing number abilities on entry to first grade? If so, what is the nature of those differences?

Approximately 1,200 entering first grade students were individually interviewed during the first six weeks of school, Fall 1979. They were partitioned into three sets based on their performance on the interview number tasks. Trios composed of a high number performance (HNP) student, intermediate number performance (INP) student, and low number performance (LNP) student were formed, with each member of the trio coming from the same first grade classroom and each being of the same gender. Thirty-eight such trios were formed, and constituted the population sample for the study.

For purposes of the study, school arithmetic was composed of addition and subtraction computational tasks, word problems, and tasks that elicited students' thinking about properties of the addition and subtraction operations. Both quantitative and qualitative assessments of student performance were carried out in describing arithmetic development across the duration of the study.

Descriptions of the school programs experienced by the students in the study were formed from information gathered from the students' classroom teachers each year.

Students in the population sample were individually interviewed during the 1980-81 school year. The interviews were composed of the tasks used to assess arithmetic development. At the end of the school year, classroom teachers were asked to respond to a set of questions formulated to describe the arithmetic program experienced by each student in the study. They were also asked to give their perception of the students' maturity level. The student interview and teacher questionnaire were re-administered during the 1981-82 school year.

The study is presented in six major parts:

1. Baseline descriptions of the study samples
2. School Arithmetic Development: Adding and Subtracting Whole Numbers
3. School Arithmetic Development: Word Problems

4. School Arithmetic Development: Commutative and Identity Properties

5. Arithmetic Program Descriptions

6. Conclusions and Implications

1. The First Grade: Baseline Descriptions of the Subjects

The following presents baseline descriptions of subjects at the first grade level whose subsequent school arithmetic development was the essential thrust of this study.

Two sets of descriptive data are presented. One set describes the performance of the thirty eight trios of students that composed the original sample population chosen in the Fall of 1979 on their entry into first grade. This group reflected a cross-section of entering first-grade students from the school district. The trios were formed on the basis of their number performance on entry to first grade, each member of the trio coming from the same classroom and experiencing the same teacher during the first grade, and each being of the same gender. The second set of data presents the performance descriptions of that subset of intact trios of students who remained in the study through Spring 1982. As expected, there was significant attrition of students composing the trios over the duration of the longitudinal study. When any one of the students from a trio was lost, the entire trio was discontinued from participation. Only trios remaining intact over the duration of the study were used in describing school arithmetic development. Hence, it was considered important to see if the baseline profiles of the subset of trios who remained throughout the duration of the study were similar to the baseline profiles of the original sample of thirty eight trios.

In the tables that follow the original thirty eight trios are designated as the "Population Sample"; the subset of intact trios retained throughout the study is designated as the "Study Sample".

Performance on Certain Specified Number Tasks on Entry to First Grade

During the first six weeks of the school year, first grade students were individually interviewed. The interview included rote and rational counting tasks as well as tasks requiring reading and writing numbers. Tables 1 - 7 present data

on responses of students in the HNP, INP, AND LNP groups to these tasks.

Students were asked to show the examiner how far they could count. They were stopped at one hundred. Table 1 shows the percentage of students that either terminated the count or made an initial error in counting ranges between 0 and 100.

Table 1

Percent of Sample Terminating Count in a Decade Range

		0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
HNP GROUP	Population Sample(N=38)	0%	0%	0%	0%	0%	0%	2%	2%	3%	92%
	Study Sample (N=20)	0%	0%	0%	0%	0%	0%	0%	0%	5%	94%
INP GROUP	Population Sample(N=38)	0%	0%	29%	52%	8%	8%	2%	0%	0%	0%
	Study Sample (N=20)	0%	0%	25%	60%	10%	5%	0%	0%	0%	0%
LNP GROUP	Population Sample(N=38)	13%	61%	26%	0%	0%	0%	0%	0%	0%	0%
	Study Sample (N=20)	5%	68%	26%	0%	0%	0%	0%	0%	0%	0%

Students in the HNP group could generally count to 100 on entry to first grade. Students in the INP group typically counted into the 31-40 range. Students in the LNP group typically could only count into the 11-20 range. There appeared to be little difference between the original population sample and the retained study sample on this counting task.

Students were asked to count backward (or down) from 5, 10 and 13. Table 2 shows the percentage of students that were able to successfully perform those tasks.

Table 2

Percent of Sample Able to Count Back from Given Number

		5	10	13
HNP GROUP	Population Sample N=38	100%	98%	79%
	Study Sample N=20	100%	100%	88%
INP GROUP	Population Sample N=38	74%	53%	3%
	Study Sample N=20	85%	55%	5%
LNP GROUP	Population Sample N=38	16%	5%	0%
	Study Sample N=20	16%	0%	0%

About all students in the HNP group could count back from 5 and 10; most were

also able to count back from 13. The majority of students in the INP group could count back from 5 and 10, but very few could count back from 13. Only a few students in the LNP group could count back from 5 and 10; none could count back from 13. There appeared to be little difference between the original population sample and the retained study sample on this counting task.

A pile of counting blocks was placed on a surface in front of the students. The examiner then asked the student to show 4, 8, 10 and 13 blocks. Table 3 shows the percentage of students able to successfully perform these tasks.

Table 3

Percent of Sample Able to Demonstrate Given Number

		4	8	10	13
HNP GROUP	Population Sample N=38	100%	97%	100%	95%
	Study Sample N=20	100%	100%	100%	94%
INP GROUP	Population Sample N=38	89%	95%	97%	74%
	Study Sample N=20	100%	100%	95%	75%
LNP GROUP	Population Sample N=38	89%	73%	47%	24%
	Study Sample N=20	89%	60%	53%	21%

Almost all students in the HNP group were able to count out 4, 8, 10 and 13 blocks on request. About all students in the INP group could count out 4, 8 and 10 blocks, and about three-fourths could count out 13 blocks. The majority of students in the LNP group could count out 4, 8 and 10 blocks, but less than one fourth could count out 13 blocks. There appeared to be little difference between the original population sample and the retained study sample on this counting task.

Students were presented with trios of dot cards with N, N + 1, N-1 dots. The trio of cards was shuffled to assure a random display. The students were then asked to identify the N card in the trio. There were four of these tasks presented where

N=3, N=6, N=10, and N=13. Table 4 shows the percentage of students able to discriminate and identify the N card in a trio of cards displayed.

Table 4

Percent of Sample Able to Discriminate Given Number from Trio

		3	6	10	13
HNP GROUP	Population Sample N=38	100%	97%	100%	95%
	Study Sample N=20	100%	100%	84%	62%
INP GROUP	Population Sample N=38	100%	82%	76%	50%
	Study Sample N=20	100%	75%	75%	45%
LNP GROUP	Population Sample N=38	92%	61%	34%	16%
	Study Sample N=20	94%	68%	42%	16%

About all students in the HNP group were able to identify the 3-, 6-, and 10-dot card in a trio of cards; the majority could identify the 13-dot card. All students in the INP group were able to identify the 3-dot card, about three fourths could identify the 6- and 10-dot cards, and about half the 13-dot card. About all of the students in the LNP group could identify the 3-dot card, slightly more than half the 6-dot card, less than half the 10-dot card, and few could identify the 13-dot card. There appeared to be little difference between the original population sample and the retained study sample on this number task.

Students were given a set of seven dot cards (one-seven) and asked to order them from least to greatest. They were then given a set of seven numeral cards (1-7) and asked to order them from least to greatest. Table 5 shows the percentage of students able to successfully perform these two tasks.

Table 5

Percent of Sample Able to Order Array and Numeral Cards

		Array	Numeral
HNP GROUP	Population Sample N=38	100%	100%
	Study Sample N=20	100%	100%
INP GROUP	Population Sample N=38	79%	89%
	Study Sample N=20	90%	95%
LNP GROUP	Population Sample N=38	37%	42%
	Study Sample N=20	32%	57%

All students in the HNP group were able to order both the array and numeral cards from least to greatest. Most students in the INP group were able to order both sets of cards. Generally less than half of the LNP group could order the two sets of cards from least to greatest. Although there were no major differences between the original population sample and the retained sample, the INP group study sample appeared to reflect a slightly higher level of ability than the original population sample.

Students were given a paper with numerals on it and asked to read each numeral. Table 6 shows the percentage of students able to correctly read each of the numerals.

Table 6

Percent of Sample Able to Read Given Numerals Correctly

		1	2	3	6	7	9	10	15	18
HNP GROUP	Population Sample N=38	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Study Sample N=20	100%	100%	100%	100%	100%	100%	100%	100%	100%
INP GROUP	Population Sample N=38	100%	100%	100%	98%	98%	84%	95%	16%	34%
	Study Sample N=20	100%	100%	100%	100%	100%	85%	100%	10%	35%
LNP GROUP	Population Sample N=38	97%	84%	89%	66%	42%	34%	45%	3%	3%
	Study Sample N=20	95%	79%	95%	63%	42%	42%	42%	5%	5%

All students in the HNP group could read all the numerals correctly. Most students in the INP group could read the numerals to 10, but few could read the teen numerals. Students in the LNP group could generally read the numerals 1, 2, 3, but then tailed-off markedly with very few able to read the teen numerals. There appeared to be little difference between the original population sample and the retained study sample in ability to read given numerals.

Students were given paper and pencil and asked to write numbers presented orally by the examiner. Table 7 shows the percentage of students able to correctly write each of the numbers.

Table 7

Percent of Sample Able to Write Given Numbers Correctly

		3	4	6	8	9	10	11	15	24
HNP GROUP	Population Sample N=38	84%	87%	82%	100%	92%	84%	100%	89%	66%
	Study Sample N=20	83%	89%	89%	100%	83%	100%	100%	94%	61%
INP GROUP	Population Sample N=38	55%	84%	68%	84%	58%	79%	66%	16%	0%
	Study Sample N=20	45%	80%	75%	85%	55%	75%	60%	20%	0%
LNP GROUP	Population Sample N=38	42%	42%	26%	32%	18%	29%	21%	0%	0%
	Study Sample N=20	32%	37%	26%	36%	26%	26%	21%	0%	0%

Most students in the HNP group were able to write all the numerals correctly. The majority of students in the INP group could write numerals to 11, but few could write 15, and none could write 24. Generally only about one fourth of the LNP group could write the numerals to 11, and none could write 15 or 24. There appeared to be little difference between the original population sample and the retained study sample in ability to write given numerals.

In summary, the HNP group chosen for the study of school arithmetic development displayed a high level of performance on rote and rational counting tasks, as

well as on reading and writing numeral tasks, on entry into first grade. The LNP group chosen for the study displayed a low level of performance on the tasks, on entry to first grade. And the INP group's performance was generally somewhere between the performance of the HNP and LNP groups on the number and numeral tasks.

A comparison of the performance of the original population sample chosen for the study of school arithmetic development and the performance of that subset of trios of students retained over the duration of the study would indicate that there was no selectivity factor at work. It appeared that the retained study sample reflected the same number and numeral performance characteristics as the original sample population chosen for the study.

Performance on Metropolitan Achievement Test Tasks, Spring of First Grade Experience

In the Spring of 1980, the Metropolitan Achievement Test (Primary I) was administered to all first grade students in the school system. The mathematics domain of the test was cluster analyzed into four topical areas: numeration, geometry and measurement, problem solving, and operations on whole numbers. Tables 8 to 11 present data on responses of students in the HNP, INP and LNP groups on those four sets of tasks.

The first 10 items on the MAT (Mathematics) purport to measure student performance on Numeration. Table 8 shows the percentage of students who responded correctly to the items in that cluster.

Table 8

Percent of Sample with Correct Response to Each Numeration Cluster Task

		1	2	3	4	5	6	7	8	9	10	T
HNP GROUP	Population Sample N=38	100%	87%	84%	79%	42%	74%	55%	55%	55%	74%	71%
	Study Sample N=20	100%	84%	89%	89%	42%	84%	63%	58%	47%	79%	77%
INP GROUP	Population Sample N=38	92%	71%	55%	50%	50%	52%	39%	37%	37%	50%	52%
	Study Sample N=20	95%	70%	55%	40%	20%	50%	25%	30%	30%	45%	51%
LNP GROUP	Population Sample N=38	58%	52%	55%	39%	21%	21%	37%	16%	16%	52%	37%
	Study Sample N=20	68%	52%	68%	42%	16%	10%	16%	16%	21%	63%	37%

With the exception of a few items in this numeration cluster, each group generally retained its relative level of performance on each of the items. On the total cluster of items, the groups clearly retained their relative level of performance originally determined by number performance on entry to first grade. Given the high degree of similarity between the original population sample and the retained study sample on total performance on this cluster of items, there appears to be no indication that the retained study sample was a bias subset of the original population sample.

Items 11-19 on the MAT (Mathematics) purport to measure student performance on Measurement and Geometry. Table 9 shows the percentage of students in each group who responded correctly to the items in that cluster.

Table 9

Percent of Sample with Correct Response to Each Measurement/Geometry Task

		11	12	13	14	15	16	17	18	19	T
HNP GROUP	Population Sample N=38	90%	79%	90%	84%	90%	71%	73%	50%	50%	75%
	Study Sample N=20	95%	79%	100%	89%	89%	84%	63%	58%	47%	75%
INP GROUP	Population Sample N=38	81%	78%	76%	65%	76%	65%	47%	45%	34%	63%
	Study Sample N=20	80%	75%	70%	60%	70%	45%	60%	50%	40%	61%
LNP GROUP	Population Sample N=38	66%	53%	79%	44%	47%	47%	63%	45%	18%	51%
	Study Sample N=20	84%	57%	73%	37%	47%	42%	37%	63%	15%	51%

With the exception of a few items in this measurement and geometry cluster, each group generally retained its relative level of performance on each item. Generally the range of differences between the three groups was more restricted on this cluster than the previous cluster; overall the groups performed at a higher level on this cluster. On the total cluster of items, the groups again retained

their relative level of performance originally determined by number performance on entry to first grade. There was a high degree of similarity between the original population sample and the retained study sample which would provide additional evidence against a selectivity bias in the retained sample.

Items 20-28 on the MAT (Mathematics) purport to measure Problem Solving performance. Table 10 shows the percentage of students in each group who responded correctly to the items in that cluster.

Table 10

Percent of Sample with Correct Response to Each Problem Solving Task

		20	21	22	23	24	25	26	27	28	T
HNP	Population Sample N=38	84%	90%	79%	53%	42%	84%	58%	71%	50%	67%
GROUP	Study Sample N=20	79%	95%	74%	53%	53%	89%	63%	63%	47%	68%
INP	Population Sample N=38	86%	76%	42%	68%	18%	47%	23%	36%	18%	46%
GROUP	Study Sample N=20	85%	80%	55%	25%	35%	45%	20%	35%	20%	44%
LNP	Population Sample N=38	58%	60%	37%	73%	21%	52%	34%	23%	18%	41%
GROUP	Study Sample N=20	63%	57%	52%	15%	5%	68%	15%	21%	10%	34%

The HNP group generally retained its relative level of performance on each of the problem solving cluster tasks. The difference between the INP group and the LNP group was not particularly marked on the individual items. On the total cluster of items, the groups retained their relative level of performance originally determined by number performance on entry to first grade. Again, the similarity between the population sample and the study sample on the total cluster of tasks would indicate that there was no selectivity factor at work.

Items 29-40 on the MAT (Mathematics) measure operations on whole numbers (addition and subtraction) performance. Table 11 shows the percentage of students in each group who responded correctly to the items in that cluster.

Table 11

Percent of Sample with Correct Response to Each Operations Task

		29	30	31	32	33	34	35	36	37	38	39	40	T
HNP	Population Sample N=38	92%	76%	71%	66%	76%	53%	84%	58%	53%	76%	39%	47%	66%
GROUP	Study Sample N=20	95%	79%	68%	68%	63%	58%	74%	74%	63%	74%	26%	52%	66%
INP	Population Sample N=38	76%	55%	45%	32%	53%	42%	24%	13%	11%	45%	26%	18%	36%
GROUP	Study Sample N=20	85%	35%	45%	25%	60%	30%	25%	20%	0%	35%	30%	20%	34%
LNP	Population Sample N=38	55%	55%	11%	39%	45%	26%	32%	5%	5%	37%	18%	18%	28%
GROUP	Study Sample N=20	68%	53%	26%	36%	26%	26%	36%	15%	10%	15%	26%	5%	28%

The HNP group clearly retained its relative level of performance on each of the operations cluster of tasks. The relative differences between the INP and the LNP groups on each of the items was not consistently observable. On the total cluster of items, the groups retained their relative level of performance originally determined by number performance on entry to first grade. The difference between the HNP group and the other two groups was much more marked than the difference between the INP and the LNP. Again, the similarity between the population sample and the study sample in performance on the total cluster tasks would indicate there was no selectivity factor at work.

Program Experiences During the First Grade.

Each of the three students composing an intact trio in the study had the same classroom teacher for his/her first grade school arithmetic experience, in addition to each being of the same gender. In June of the first-grade year, teachers were asked to respond to a questionnaire about aspects of the school arithmetic program experienced by each member of the trio during the first grade.

Comparisons between the original population sample and the retained study sample presented in the previous sections offered no compelling evidence of differences between these two groups. It is assumed that there was no selectivity

factor at work and the retained study sample was similar in composition to the original sample population. Ensuing data presentations are based exclusively on performances of the retained study population, N=20 trios.

Tables 12 to 14 present descriptive data on aspects of the school arithmetic program experienced by students in the HNP, INP and LNP groups during the first grade. Teachers were asked to indicate the amount of time given to school arithmetic instruction daily in the classroom, whether a basal textbook was used for instruction, how often the textbook was used, and whether it was completed during the year. Table 12 presents the results of those inquiries.

Table 12

Percent of Students Experiencing Particular Program Conditions

	Daily Classroom Arithmetic Time in Minutes			Basal Test Usage		Frequency of Textbook Usage				Textbook Completed During Year	
	15-30	30-45	45-60	Yes	No	D	3-4 Week	1-2 Week	Ir-Reg	Yes	No
HNP Group	5%	75%	20%	90%	10%	60%	30%	5%	5%	60%	40%
INP Group	20%	60%	20%	100%	0%	60%	25%	10%	5%	50%	50%
LNP Group	15%	60%	25%	85%	15%	60%	25%	10%	5%	40%	60%

The program profiles that emerge from the questionnaire administered to teachers appeared to be quite similar for students in all three groups. Typically, students received thirty to forty five minutes per day classroom instruction in arithmetic. With few exceptions, a basal textbook was used for instruction. There were a few cases in the HNP and LNP groups where the DMP program was used rather than a traditional textbook. Typically, the textbook was used daily during

the first grade experience; very seldom was it used less than three to four times per week. Only about one half of the students completed the textbook during the first grade, with a higher ratio of students in the HNP group than in the LNP group completing the textbook.

The classroom teachers were also questioned regarding their classroom organization for arithmetic instruction, and whether students received supplementary arithmetic instruction outside the classroom. Table 13 presents the results of those inquiries.

Table 13

Percent of Students Experiencing Particular Instructional Conditions

	Classroom Organization for Instruction					Supplementary Instruction		
	Total Class Instruction	Total Class w/ Small Group or Ind. Instruction	Small Group Instruction	Small Group w/Ind. Instruction	Individual Instruction	Yes	No	Min/ Week
HNP Group	35%	45%	10%	10%	0%	30%	70%	105
INP Group	35%	50%	5%	10%	0%	35%	65%	120
LNP Group	20%	45%	15%	20%	0%	70%	30%	128

Classroom organization for arithmetic instruction appeared to be quite similar for students in all three groups. There was some slight indication that the LNP group received less total class instruction and more small group and individual instruction. There were no teachers that indicated they used an exclusive individualized instruction procedure. There was a significant difference between the LNP group and the HNP and INP in students receiving supplementary instruction.

Typically, the students in the LNP group received a little more than two hours per week supplementary instruction, primarily from Title I mathematics correction teachers. Less than half of the students in the HNP and INP received supplementary instruction. In the case of the HNP students, this was typically enrichment work with ESAA mathematics personnel.

Classroom teachers were asked to order the four areas of the first grade curriculum according to the priority of importance and emphasis given each. Table 14 presents the teachers' responses regarding relative order of emphasis of these four curriculum areas with "1" indicating high priority and "4" indicating low priority of emphasis.

Table 14

Percent of Teachers Indicating Priority of Emphasis Given Four Curriculum Areas

	HNP GROUP				INP GROUP				LNP GROUP			
	Order of Emphasis				Order of Emphasis				Order of Emphasis			
	1	2	3	4	1	2	3	4	1	2	3	4
Number Development	65%	25%	10%	0%	60%	25%	15%	0%	60%	20%	15%	5%
Measurement/Geometry	0%	25%	30%	45%	0%	25%	40%	35%	0%	25%	30%	45%
Add/Sub Combinations	40%	45%	10%	5%	40%	50%	10%	0%	35%	55%	10%	0%
Word Problems(+,-)	0%	5%	40%	55%	0%	0%	35%	65%	0%	5%	45%	50%

The profile of priorities given to particular areas of the first-grade arithmetic curriculum was very similar for all three groups. Typically, top priority was given to number development activities with work on the addition and subtraction combinations being the second most important area of emphasis. Measurement and geometry activities and work with word problems appeared to be low priority areas

of emphasis. These priorities of emphasis appeared to hold across all three groups.

The teachers were asked to indicate their perceptions of each student's level of maturity on a five point scale, with "5" indicating a high level of maturity, and "1" a low level of maturity. Four different areas of maturity were presented to the teachers for response: arithmetic maturity, intellectual maturity, social/emotional maturity, and psycho/motor maturity. Table 15 presents the teachers' responses for each of the three groups.

Table 15

Percent of Teachers Indicating Perceived Levels of Maturity

	HNP GROUP				INP GROUP				LNP GROUP			
	Arith Mat.	Int. Mat.	Soc/Em Mat.	Psy/M Mat.	Arith Mat.	Int. Mat.	Soc/Em Mat.	Psy/M Mat.	Arith Mat.	Int. Mat.	Soc/Em Mat.	Psy/M Mat.
LEVELS 1	0%	0%	0%	0%	10%	10%	15%	10%	50%	30%	35%	25%
2	0%	0%	5%	0%	25%	35%	25%	30%	40%	55%	40%	30%
3	10%	10%	15%	10%	50%	40%	35%	30%	10%	15%	20%	40%
4	35%	30%	15%	30%	15%	15%	25%	30%	0%	0%	5%	5%
5	55%	60%	65%	60%	0%	0%	0%	0%	0%	0%	0%	0%

The three groups differed significantly on teachers' perceptions of their maturity in the four areas. Students in the HNP group were perceived by their teachers as being quite mature on all four dimensions. Typically, they were judged to be "4" or "5" on a five-point scale in all areas of maturity. Students in the LNP group were perceived by their teachers as being quite immature on all four dimensions. Typically, they were judged to be "1" or "2" on the scales. The INP group typically was judged to be "2", "3", or "4" on the five-point scale. It appeared that teachers' judgment of students' maturity at the end of first grade was very closely related to the students' performance on the number tasks

on entry to first grade, as well as performance on the MAT (Mathematics) during the first-grade experience.

This section of the report has presented baseline descriptions of subjects to be studied and the school staging conditions from which the study of school arithmetic development ensued over the subsequent two-year period. All subjects in the study entered first grade in Fall 1979. On the basis of performance on interviews of about 1,200 incoming first graders, three groups of students were formed: a high number performance group (HNP), a low number performance group (LNP), and an intermediate number performance group (INP). Thirty eight trios of students composed the initial population sample. One number of each trio was in the HNP group, one in the INP group, and one in the LNP group. Each member of a trio was of the same gender, and each was in the same classroom with the same teacher during the first-grade experience. Twenty intact trios were retained over the duration of the study. Data that compared the original population sample (38 trios) with the retained study sample (20 trios) indicated few differences in performance between the two groups. Hence, it appeared reasonable to assume that the retained study sample reflected a cross-section of students from the large urban school district from which the original population sample was drawn.

To summarize the characteristics of the three groups of students in the study, on entry to first grade HNP group students could typically count to one hundred; could count back from 5, 10 and 13; could demonstrate 4, 8, 10 and 13 blocks; could discriminate 3, 6, 10 and 13 dot cards; could order array and numeral cards 1 through 7; could read numerals through the teens; and could write numbers through the teens. LNP group students could typically not count beyond 20; could not count back from 5, 10 or 13; could not demonstrate a number of blocks beyond 10; could not discriminate dot cards beyond 10; had difficulty ordering array and numeral cards 1 through 7; could not read numerals beyond 10; and had difficulty writing numerals to 10.

The number performance of the INP group on entry to first grade was generally between that of the HNP and LNP groups on each of the number tasks.

In performance on the Metropolitan Achievement Test (Mathematics) administered in the Spring of the first-grade year, the three groups retained their same relative level of performance on each of the content clusters. The HNP group responded correctly to about 75% of the tasks on the Numeration cluster, about 75% of the tasks on the Measurement/Geometry cluster, 67% of the tasks on the Problem Solving cluster, and 66% of the tasks on the Operations cluster. The LNP group responded correctly to about 37% of the tasks on the Numeration cluster, 51% of the tasks on the Measurement/Geometry cluster, 35% of the tasks on the Problem Solving cluster, and 28% of the tasks on the Operations cluster. The performance of the INP group was between that of the HNP and LNP groups on each of the clusters.

All three groups received about the same classroom instruction time during the first-grade experience, but the LNP group received significantly more supplementary instruction outside the classroom than either of the other two groups. Students in all three groups used textbooks in their first-grade experience, and the frequency of use was about the same for all groups. More students in the HNP group completed the text during the first grade than did LNP group students. The classroom organization provided for instruction was very similar for all students. The emphasis given to first-grade arithmetic topics by teachers was very similar for students in all groups.

Their first-grade teachers tended to perceive students in the LNP group as being very immature in all areas of development. They perceived the HNP group of students as being very mature in all areas of development. Teacher perceptions of the maturity of the INP group tended to fall between the two extremes.

How then do these three groups of students who have marked differences in number performance on entry to first grade, who differ correspondingly in their

performance on achievement test tasks administered during the first grade, and who have correspondingly marked differences in maturity in the perceptions of their first-grade teachers, progress in arithmetic development during the two subsequent years of their school experience? The following sections of this report present quantitative and qualitative information about aspects of their arithmetic development.

2. School Arithmetic Development: Adding and Subtracting Whole Numbers

Students in each of the three groups were administered twelve addition and subtraction tasks as part of the individual interviews carried out each year over the two-year duration of the study. The 1981 interviews took place during the second half of the students' second year of school beyond kindergarten; the 1982 interviews took place during the second half of their third year of school. Figure 1. shows the twelve addition and subtraction tasks. The first ten tasks required no renaming; the final two tasks involved three digit numbers where renaming was involved and were tasks that went beyond the typical first grade curriculum.

Students were given the individual sheet and directed to complete as many tasks as they could. Manipulative material in the form of unit-cubes and ten-rods was available to each student, and they were informed that they could use the materials, or not, as they wished. The interviewers observed each student respond to the tasks and recorded observations of overt physical manifestations of processing displayed by each subject. Both quantitative and qualitative analyses of the data were carried out.

Quantitative Analysis: The mean and standard deviation of performance scores for the HNP, INP, and LNP groups for both the 1981 and 1982 administrations are shown in Table 16. Group scores on the twelve tasks administered in 1981 directly corresponded with the performance of the groups on the number tasks on entry to first grade in 1979. The corresponding performance of the groups continued through the 1982 administration. The mean of each group increased by a similar increment in performance between the 1981-82 administration, although the HNP group increase was somewhat restricted because of the ceiling effect of the test.

NAME _____ GRADE _____

School _____ Teacher _____

① $\begin{array}{r} 3 \\ +6 \\ \hline \end{array}$ ② 7 + 8 = ___ ③ 9 - 6 = ___ ④ $\begin{array}{r} 15 \\ -\ 8 \\ \hline \end{array}$

⑤ 4 + 5 + 8 = ___.

⑥ $\begin{array}{r} 3 \\ 6 \\ +\ 7 \\ \hline \end{array}$

⑦ 21 + 6 = ___ ⑧ $\begin{array}{r} 23 \\ +\ 14 \\ \hline \end{array}$ ⑨ 27 - 5 = ___ ⑩ $\begin{array}{r} 38 \\ -15 \\ \hline \end{array}$

⑪ $\begin{array}{r} 146 \\ +\ 271 \\ \hline \end{array}$ ⑫ $\begin{array}{r} 418 \\ -\ 172 \\ \hline \end{array}$

Figure 1. Twelve Addition and Subtraction Tasks

Table 16

Group Performance on Addition and Subtraction Tasks

	Spring 1981		Spring 1982	
	$\bar{X}$	Sd.	$\bar{X}$	Sd.
HNP Group	9.80	1.44	11.25	1.25
INP Group	7.65	1.76	9.71	1.87
LNP Group	5.25	2.47	7.60	2.98

The ogives of the three groups' performance on the twelve tasks shown in Figure 2 gives a further more refined perspective on the performance of the students in 1981 and 1982. It can be noted that students in the LNP group with scores above the median on the addition and subtraction tasks generally performed within the same range of scores as students in the HNP and INP groups. Students in the LNP group with scores below the median performed below the entire range of scores of students in the HNP and most students in the INP group.

About fifty percent of the students in the LNP group on entry to first grade had performance scores on the addition and subtraction tasks that were below the entire range of performance scores of the HNP group. Their scores were also below about ninety percent of the performance scores of the INP group. This was generally true for the 1981 and 1982 administrations of the tasks. The fifty percent of students at or above the median in the LNP group had performance scores that fell within the range of scores of the HNP and INP groups at the time of the second 1982 administration.

Table 17 shows the performance of students in the three groups on each of the twelve addition and subtraction tasks. Most of the improvement in performance between the 1981/82 administration for the HNP group is attributable to the last

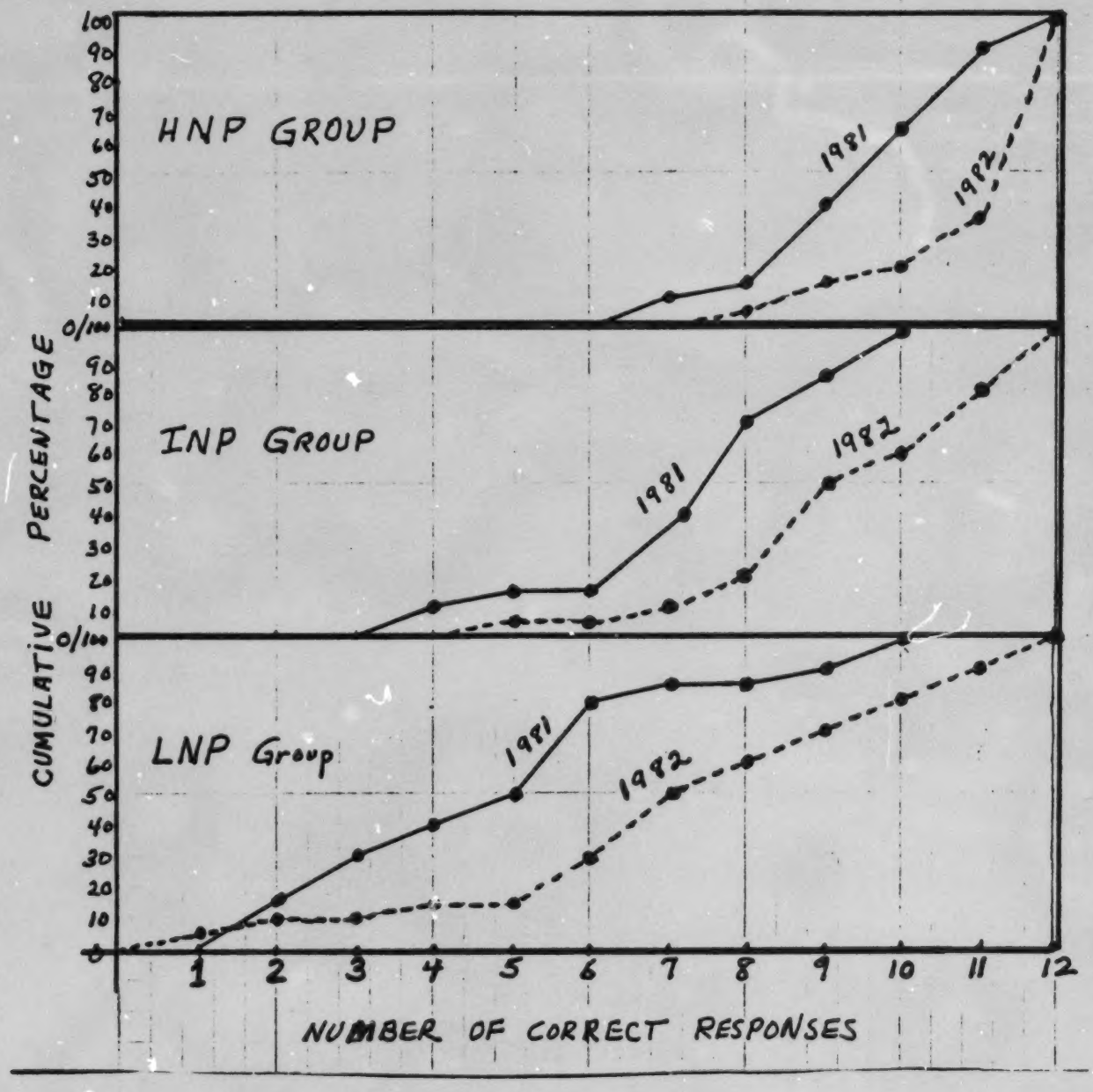


Figure 2. Ogives of Performance on Twelve Computation Tasks

two tasks involving renaming. Both the INP and the LNP generally improved performance on all the tasks. The last two tasks appeared to be difficult for these groups with less than fifty percent getting them correct. The two more difficult addition and subtraction tasks presented in horizontal form (21 + 6, and 27 - 5) were especially troublesome to the LNP group, with only about fifty percent being able to respond correctly in the 1982 administration.

Table 17

Percentage of Students in Each Group Responding Correctly to computation Tasks

Item		1	2	3	4	5	6	7	8	9	10	11	12
HNP	1981	100%	100%	95%	85%	95%	95%	80%	95%	70%	80%	50%	20%
Group	1982	100%	100%	100%	95%	85%	95%	90%	95%	95%	100%	90%	80%
INP	1981	100%	85%	90%	70%	75%	65%	50%	80%	40%	60%	15%	0%
Group	1982	100%	95%	95%	80%	100%	80%	80%	85%	75%	85%	55%	40%
LNP	1981	85%	70%	75%	65%	40%	50%	35%	40%	40%	20%	5%	0%
Group	1982	90%	60%	80%	85%	75%	75%	50%	80%	45%	65%	40%	15%

An error analysis of the twelve items was carried out to gain further insight into the development of the three groups of students. A three-by-three matrix was constructed for each item and each group. (See figure at right.)

The nine cells in the matrix indicate: (A) the number responding correctly to the item in both the 1981 and 1982 administration; (B) the number responding correctly in 1981 but incorrectly in 1982; (C) the number responding correctly in 1981 but not attempting in 1982; (D) the number responding incorrectly in 1981 but correctly in 1982; (E) the number responding incorrectly in both 1981 and 1982; (F) the number responding incorrectly in 1981 and not attempt-

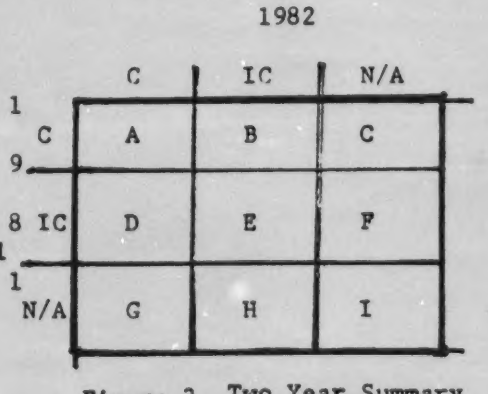
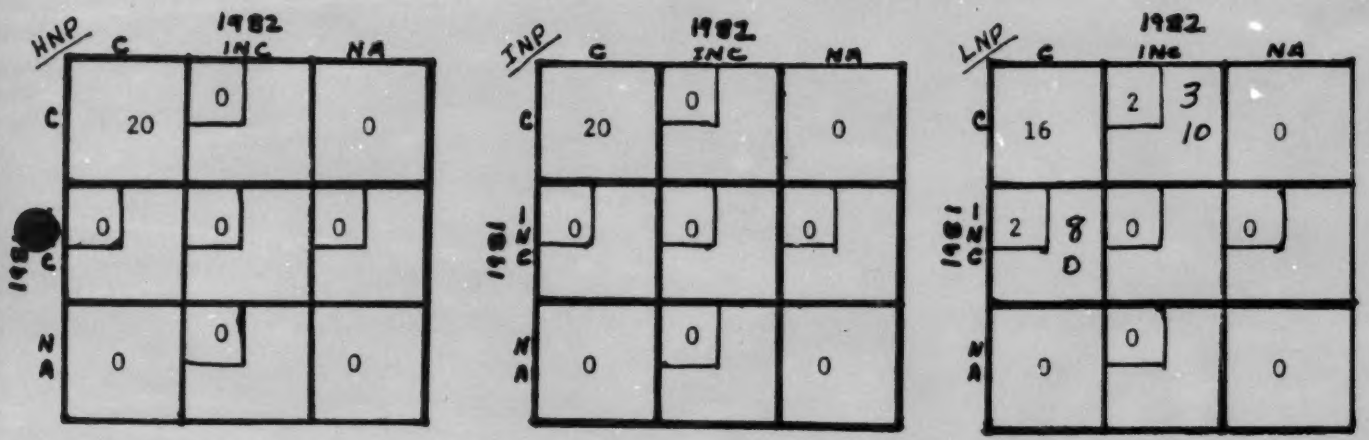


Figure 3. Two Year Summary Matrix

ing the task in 1982; (G) the number not attempting in 1981 and responding correctly in 1982; (H) the number not attempting in 1981 and responding incorrectly in 1982; (I) the number not attempting in either 1981 or 1982. In cells B, D, E, F and H the frequency for the cells is recorded in the inner cell, and the incorrect responses in the remaining area.

Table 18

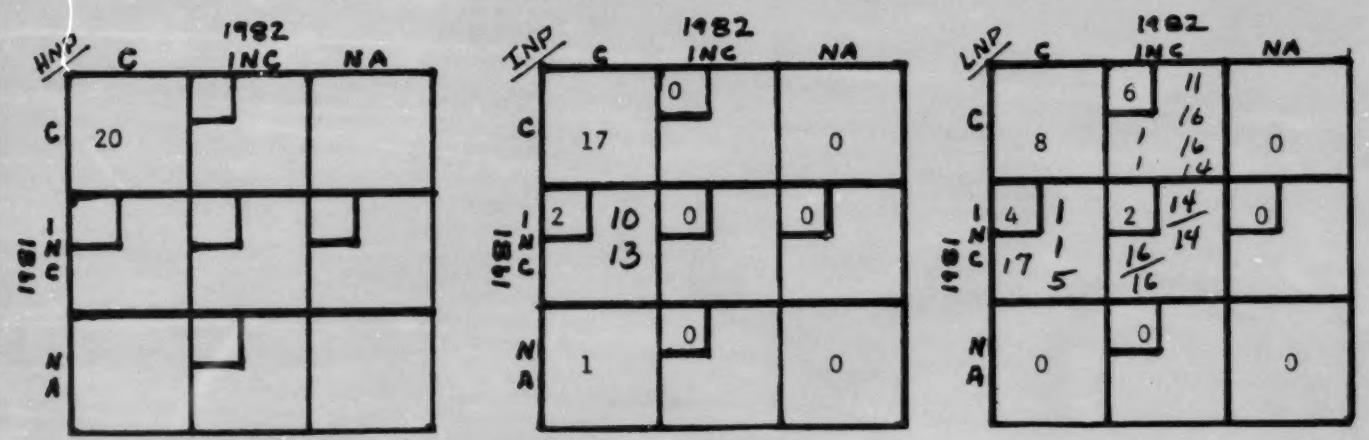
Summary of Individual Responses to Item #1, 1981/82 Administrations



Item #1 was answered correctly by all students in the HNP and INP groups in both years. Two students in the LNP group were incorrect in 1981 but correct in 1982; two were correct in 1981 but incorrect in 1982. Two of the incorrect responses were adjacent numbers to 9 (8,10); another incorrect response was 3, probably indicating choice of incorrect operation (subtract). The 0 response is difficult to understand.

Table 19

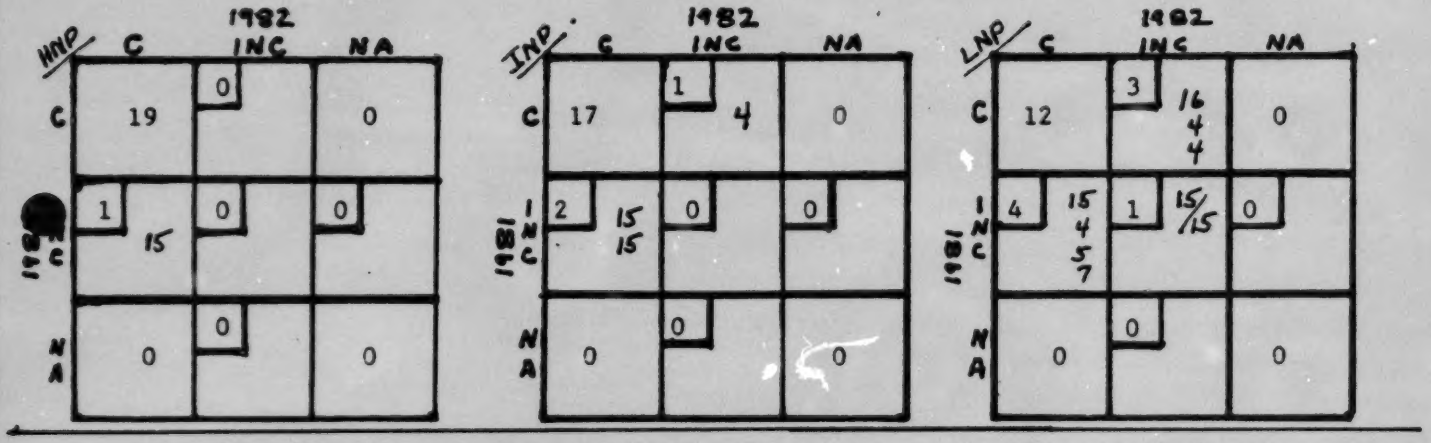
Summary of Individual Responses to Item #2, 1981/82 Administrations



Item #2 was answered correctly by all students in the HNP group both years. Two studentd in the INP group were incorrect the first year, but all were correct the second year. The incorrect responses "10" and "13" were difficult to interpret. Only eight students in the LNP group had the task correct both years. Two students had the task incorrect both years and were consistent in their errors; one gave 14 as a response both years, the other 16 both years. Ten students had incorrect answers either in 1981 or 1982 but not both; four were incorrect in 1981 but correct in 1982; six were correct in 1981 but incorrect in 1982. Most common errors were "1", where students evidently were performing the wrong operation (subtraction), and "14" and "16", adjacent numbers to the correct response 15. Reasons for the other incorrect answers such as 5 and 17 were less evident.

Table 20

Summary of Individual Responses to Item #3, 1981/82 Administrations

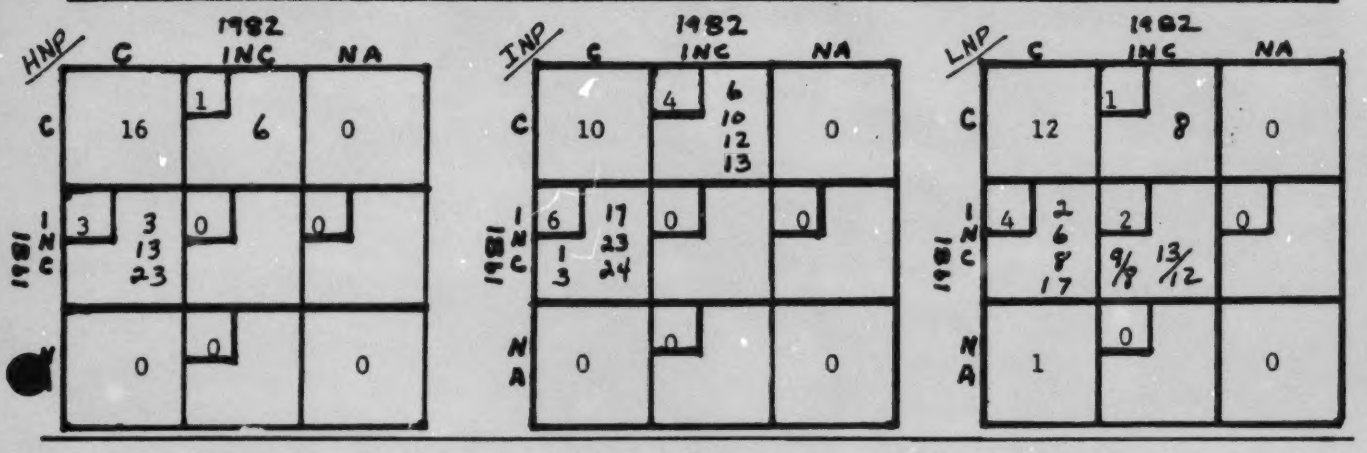


Item #3 was answered correctly by all students in the HNP group both years except in one case where a student responded incorrectly in the 1981 administration. That case evidently involved incorrect choice of operation. Most INP group students responded correctly both years. Two gave incorrect responses in 1981, but were

correct in 1982; one gave a correct response in 1981, but was incorrect in 1982. Both students in the former case responded "15" and evidently chose the incorrect operation; in the latter case the incorrect response was "4", an adjacent number to the correct response 3. The LNP group had a number of incorrect responses; four were incorrect in 1981 but correct in 1982; three were correct in 1981 but incorrect in 1982; and one was incorrect both years. The most common incorrect responses were "4", ad adjacent number to the correct response 3, and "15" where students evidently chose the incorrect operation (addition). Reasons for other responses such as "16", "5" and "7" were less evident.

Table 21

Summary of Individual Responses to Item #4, 1981/82 Administrations

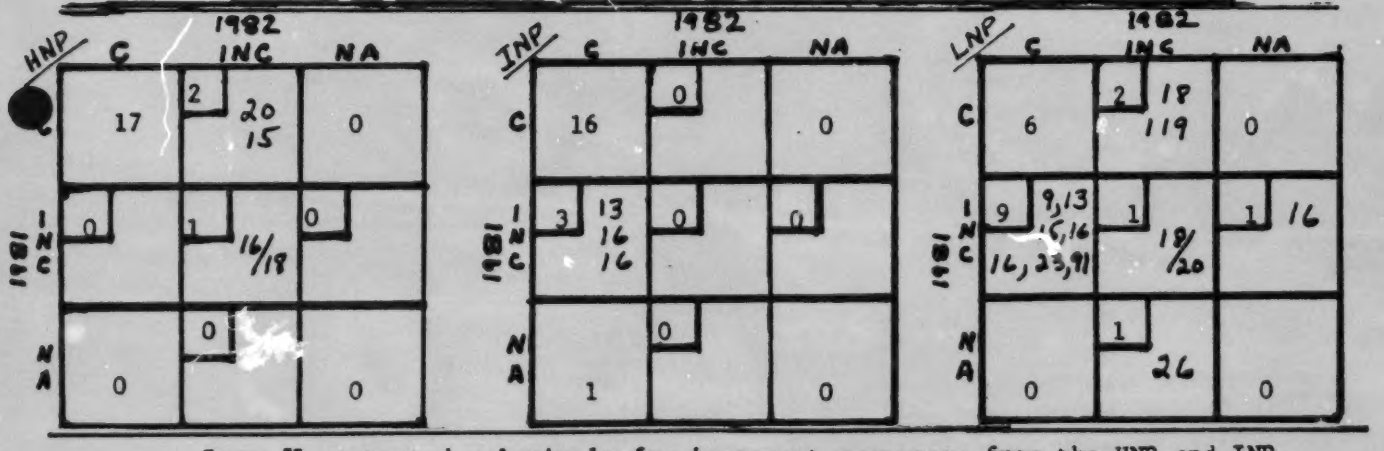


Item #4 elicited three incorrect responses in 1981 from the HNP group but those three students responded correctly in 1982; one student on the HNP group responded correctly in 1981 but incorrectly the next year. The three incorrect responses of the former group: "3", "13", and "23" all indicated the students were subtracting five from 8 as a procedure, or choosing the incorrect operation (addition) in the task. The other incorrect response was "6", an adjacent number to the correct response 7. Only ten students in the INP group responded correctly both years.

Six students responded incorrectly the first year, but incorrectly the second. Incorrect responses included "3", "13", "23" and "6" as in the previous group. Other responses such as "1", "10" and "12" were difficult to interpret. The incorrect response "24" was an adjacent number if the incorrect operation of addition was carried out; the incorrect "17" probably involved the student "bringing down" the 1 in "15" after subtracting 8 from 15. In the LNP group, four students responded incorrectly in 1981 but correctly in 1982, while one responded correctly in 1981 but incorrectly in 1982. Two students responded incorrectly both years to the task, and one did not attempt the task in 1981 and responded correctly in 1982. Three of the incorrect responses were "8" and one was "6", all adjacent numbers to the correct response 7. Other incorrect responses included "12" and "17" which were responses made by students in the other groups. The incorrect responses "2", "9", and "12" were not made by other groups and are difficult to understand.

Table 23

Summary of Individual Responses to Item #5, 1981/82 Administrations

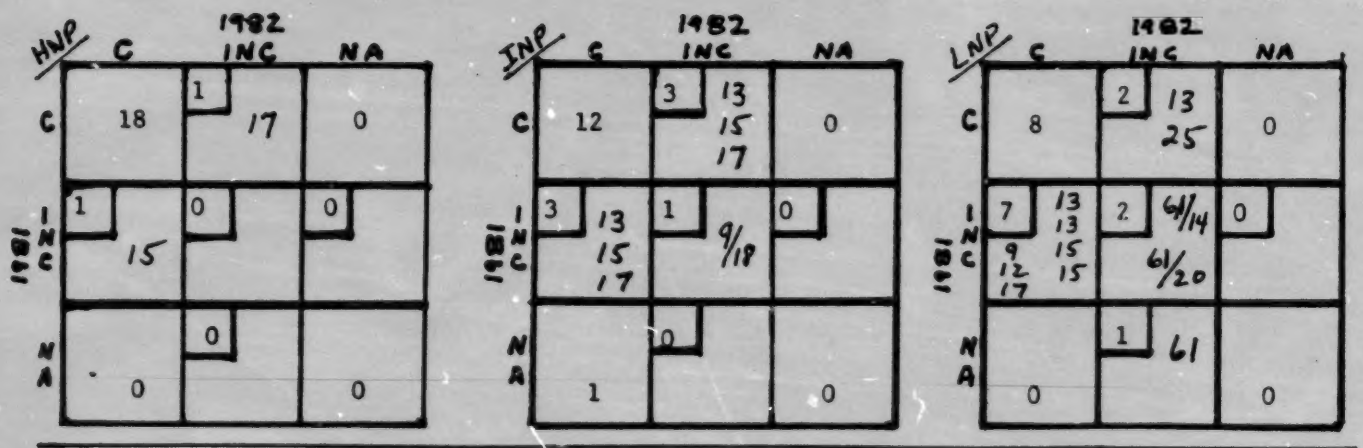


Item #5 generated relatively few incorrect responses from the HNP and INP groups. Two students in the HNP group responded correctly in 1981 but incorrectly in 1982, while one responded incorrectly both years. In the latter case, the student responded "16" and "18", both adjacent to the correct answer 17. The

other two incorrect answers, "20" and "15" are more difficult to interpret. In the INP group, three students responded incorrectly in 1981 but correctly in 1982; one student made no attempt in 1981 but responded correctly in 1982. Of the three incorrect responses, two were "16", a number adjacent to the correct response 17. The "13" response could have been the result of the student ignoring the 4 in the task and simply responding to 5 + 8. This task was especially difficult for the LNP group at the time of the 1981 administration; by the 1982 administration, there was significant improvement. Nine students responded incorrectly in 1981 but correctly in 1982; two responded correctly in 1981 but incorrectly in 1982; one responded incorrectly both years; one responded incorrectly in 1981 and did not attempt in 1982; one did not attempt in 1981 and responded incorrectly in 1982. Seven of the incorrect responses given were either "16" or "18", adjacent numbers to the correct response 17. Two incorrect responses were "9" and "13", again the student evidently choosing a recognizable two addend combination from the three presented. Other incorrect responses "15", "20", "23", "26", "91", and "119", are difficult to interpret.

Table 23

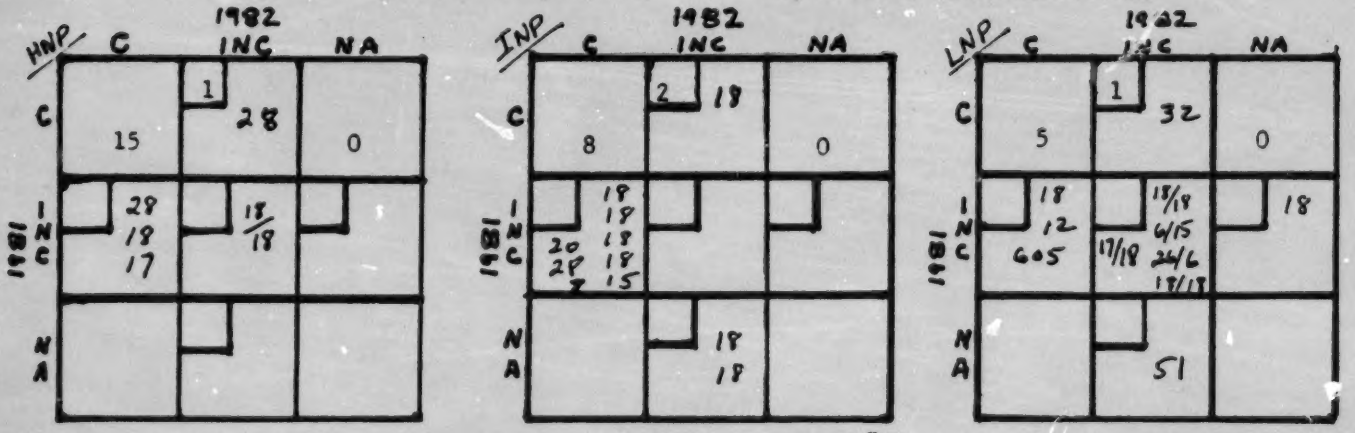
Summary of Individual Responses to Item #6, 1981/82 Administrations



Item #6 was quite easy for the HNP group; one student responded incorrectly in 1981 but correctly in 1982; the other responded correctly in 1981 but incorrectly in 1982. Both incorrect responses, "15" and "17" were adjacent to the correct response of 16. In the INP group three students responded incorrectly in 1981 but correctly in 1982; three responded correctly in 1981 but incorrectly in 1982; one student responded incorrectly both years; one did not attempt in 1981 and responded correctly in 1982. Four of the incorrect responses were "17" and "15", adjacent numbers to the correct response of 16. Two of the incorrect responses were "13", where the students evidently added the two addends 6 and 7 and forgot or ignored the addend 3; another incorrect response was "9", again the student evidently responded to 3 plus 6 and forgot or ignored the 7. The other incorrect response of "18" was more difficult to interpret. In the LNP group, seven students responded incorrectly in 1981 but correctly in 1982; two responded correctly in 1981 but incorrectly in 1982; two responded incorrectly both years; one did not attempt in 1981 and responded incorrectly in 1982. Three of the incorrect answers were either "15" or "17", numbers adjacent to the correct answer of 16. Four incorrect responses were either "13" or "9" where the students evidently responded to only two of the three addends. Three of the incorrect responses were "61" where students evidently reversed 16 in recording their responses. Other incorrect responses, "12", "14", "20", and "25" were difficult to interpret.

Table 24

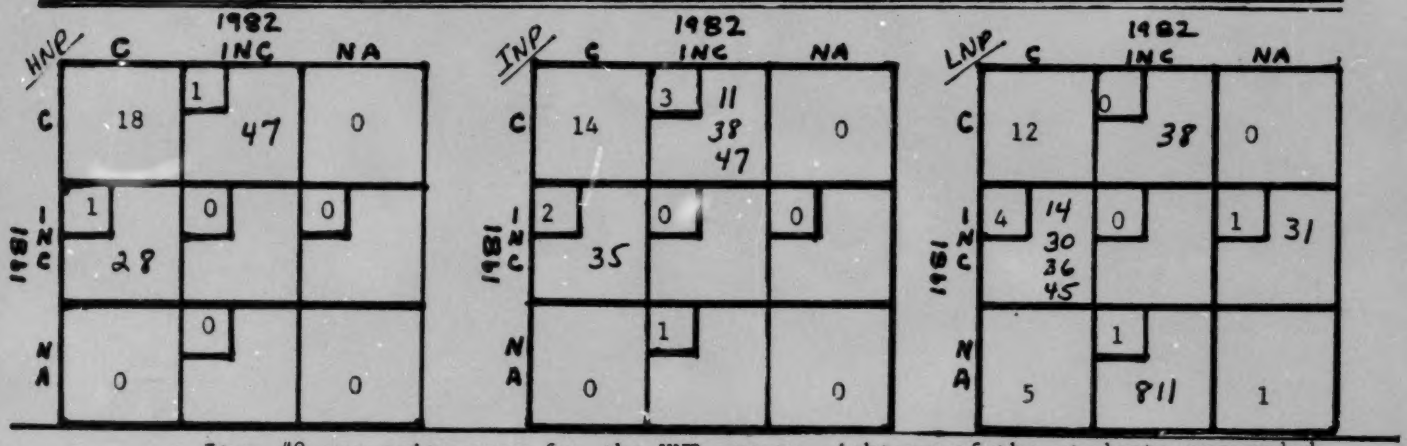
Summary of Individual Responses to Item #7, 1981/82 Administrations



Item #7 was quite easy for the HNP group. Three students responded incorrectly in 1981 but correctly in 1982; one responded correctly in 1981 but incorrectly in 1982; and one responded incorrectly both years. Most of the incorrect responses were "18" and "28". In the latter case the incorrect response was an adjacent number to the correct response of 27; in the former case the students evidently added the 6 and 2 and recorded the 1 in the tens place. The reason for the incorrect response "17" is less evident. The INP group tended to respond incorrectly in the 1981 administration but correctly in 1982. Eight students responded incorrectly in 1981 but correctly in 1982; two responded correctly in 1981 but incorrectly in 1982; two did not attempt the task in 1981 and were incorrect in 1982. Eight of the incorrect responses were "18", evidently adding the 2 and 6 and recording the 1 in the tens place. One student incorrectly responded "28", an adjacent number to the correct response of 27. The incorrect response "15" evidently resulted from use of the incorrect operation, subtraction. The reasons for the incorrect responses "8" and "20" were not evident. The item was difficult for the LNP group in both administrations. Only five were correct both years; eight were incorrect both years; three were incorrect in 1981 but correct in 1982; one was correct in 1981 but incorrect in 1982; one was incorrect in 1981 and did not attempt in 1982; one did not attempt in 1981 and was incorrect in 1982; and one did not attempt in 1981 and was correct in 1982. Although "18" was the most common incorrect response, the group generated many different incorrect responses (6, 12, 13, 15, 16, 26, 51, 61, 605), where their reasoning was not evident.

Table 25

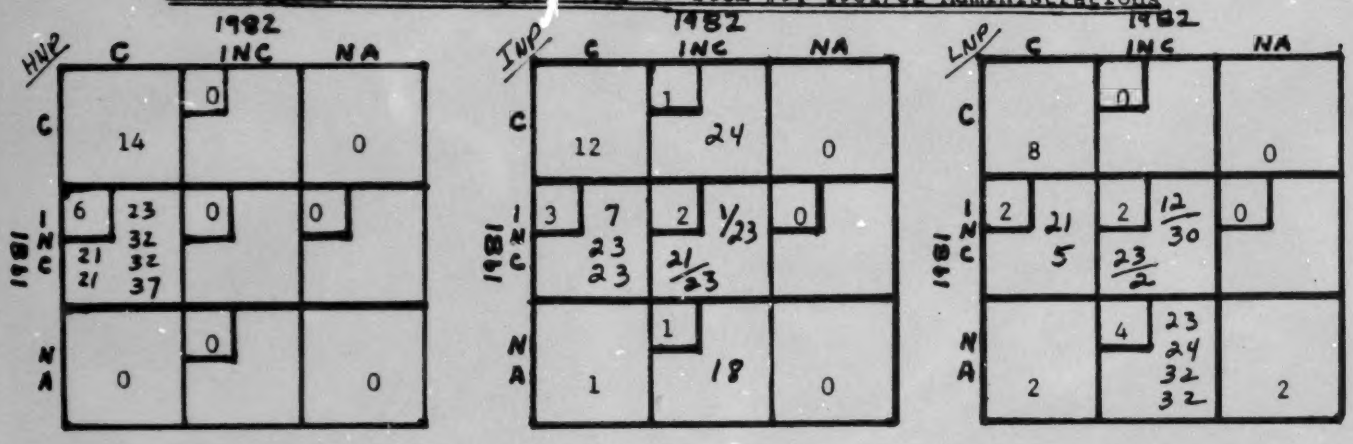
Summary of Individual Responses to Item #8, 1981/82 Administrations



Item #8 was quite easy for the HNP group; eighteen of the students responded correctly both years. One responded incorrectly the first year but correctly the second; one correctly the first but incorrectly the second. One incorrect response was "47", where the student evidently carried from ones to ten; the other incorrect answer was "28" which is difficult to interpret. The INP group had 14 students who answered correctly both years; two who answered incorrectly in 1981, but correctly in 1982; and one who did not attempt in 1981 and was incorrect in 1982. Two incorrect responses "11" and "17" would indicate the student subtracted in all or part of the processing. The response "47" indicated use of carrying when there was none. The "38" incorrect response was an adjacent number to the correct answer of 37. The "35" response is more difficult to interpret. The LNP group did relatively well on this task. Twelve had it correct both years; four were incorrect in 1981 but correct in 1982; five did not attempt in 1981 but were correct in 1982; one was incorrect in 1981 and did not attempt in 1982; one did not attempt in 1981 and was incorrect in 1982; and one did not attempt either year. The "31" incorrect response indicated partial use of the incorrect subtraction operation; the "36" incorrect response was an adjacent number to the correct response of 37. Other incorrect responses "30", "14", "45" and "811" were difficult to interpret.

Table 26

Summary of Individual Responses to Item #9, 1981/82 administrations

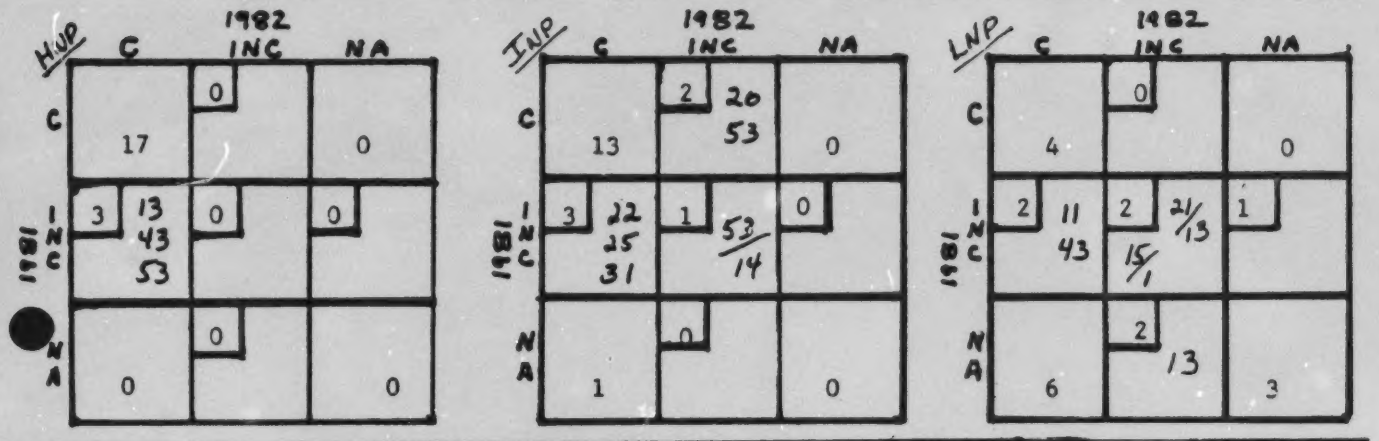


Item #9 was easy for the HNP group by the time of the 1982 administration. Fourteen were correct both years; six were incorrect in 1981 but responded correctly in 1982. Three of the incorrect responses were "21" or "23", adjacent numbers to the correct response of 22. Two of the incorrect responses were "32" where the incorrect operation was evidently applied. The "37" incorrect response evidently was the result of subtracting 5 from 2 (sic) and recording the 7 in the ones place. In the INP group, twelve students responded correctly both years; two responded incorrectly both years. Three students responded incorrectly in 1981 but correctly in 1982; one student responded correctly in 1981 but incorrectly in 1982. One student did not attempt the task in 1981 but was correct in 1982. Five of the incorrect responses were "23" or "21", adjacent numbers to the correct response of 22. The incorrect responses "1", "7", "18" and "24" are difficult to interpret. The item was difficult for the LNP group; only eight students responded correctly both years. Two students responded incorrectly both years; two students responded incorrectly in 1981 but correctly in 1982; two students did not attempt in 1981 but were correct in 1982; four did not attempt in 1981 and were incorrect in 1982; and two did not attempt either year. Three of the incorrect responses were "21" or "23", adjacent numbers to the correct response of 22. Two incorrect responses were "32" which indicated choice of the wrong operation. The incorrect

response "12" evidently indicated a "borrowing" procedure where none was needed. The other incorrect responses "2", "5" "24" and "30" were difficult to interpret.

Table 27

Summary of Individual Responses to Item #10, 1981/82 Administrations

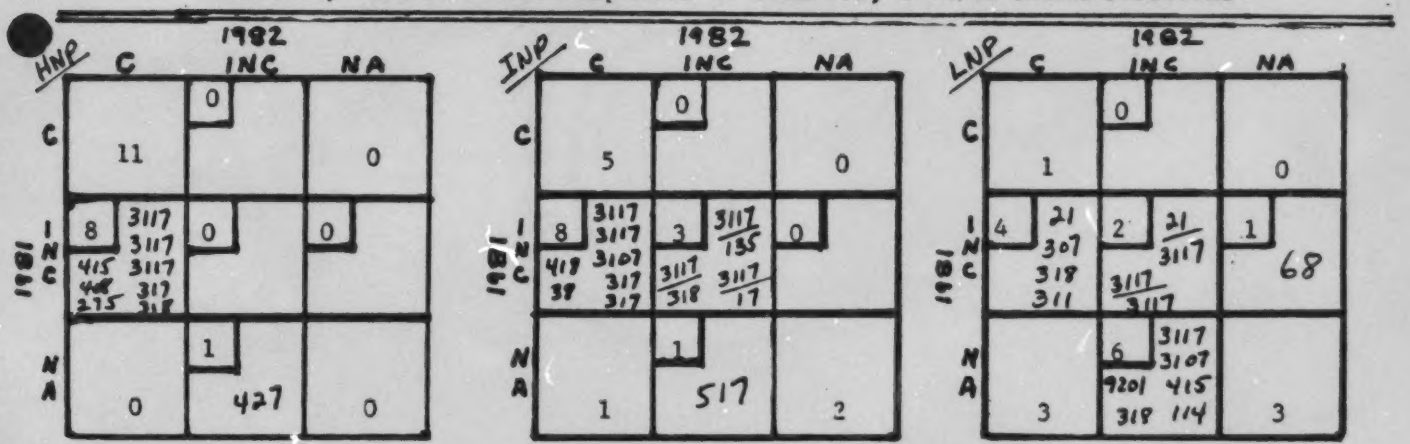


Item #10 was quite easy for the HNP group; seventeen responded correctly in both administrations. Three students responded incorrectly in 1981, but correctly in 1982. The three incorrect responses were "12, "43", and "53". Two of these evidently were choice of the incorrect operation of addition, and the other the use of a borrowing procedure where none was needed. In the INP group, thirteen responded correctly in both administrations; three responded incorrectly in 1981 but correctly in 1982; two correctly in 1981 but incorrectly in 1982; one responded incorrectly both times; and one did not respond in 1981 but responded correctly in 1982. Two of the incorrect responses were "53" and evidently involved the application of the incorrect operation, addition. One incorrect response was "22", an adjacent number to the correct response of 23. The reasons for the other incorrect responses "14", "20", "25" and "31" were not evident. Only four students in the LNP group responded correctly in both administrations; two responded incorrectly in both administrations. Two students responded incorrectly in 1981 but correctly in

in 1982; six did not attempt to respond in 1981 but were correct in 1982; two did not attempt to respond in 1981 and were incorrect in 1982; one responded incorrectly in 1981 and did not attempt in 1982; and three did not attempt either time. Two of the incorrect responses were "13" which evidently was the result of borrowing when it was not needed. The "43" incorrect response may have been the result of adding without carrying. Other incorrect responses "11", "15", "21" and "3" were more difficult to interpret.

Table 28

Summary of Individual Responses to Item #11, 1981/82 Administrations

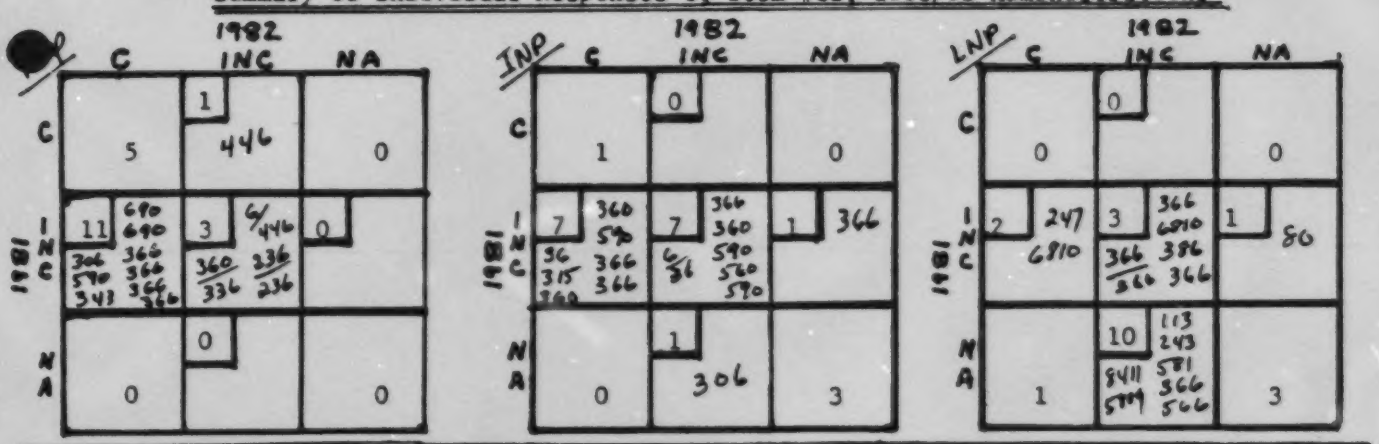


Item #11 was answered correctly by eleven students in the HNP group in both administrations. Eight responded incorrectly in 1981 but correctly in 1982; one did not attempt in 1981 and answered incorrectly in 1982. The most common incorrect responses were"3117"or"317", quite reasonable responses when formal instruction had not been carried out as yet. The "275" incorrect response was apparently a result of a mixture of addition and subtraction in processing. The "415", "408", and "427" incorrect responses were apparently some combination of incorrect carrying and confusion in adding or subtracting. Five students in the INP group responded correctly in both administrations; three responded incorrectly in both attempts.

Eight students responded incorrectly in 1981 but correctly in 1982; one did not respond in 1981 and responded correctly in 1982; one did not respond in 1981 and responded incorrectly in 1982; and two did not attempt in either administration. The common incorrect responses were again those suxh as "3117", "3107", "317", "517", "418", and "318", where errors in the carrying procedure occurred. The "135" incorrect response was obviously a result of subtracting instead of adding. Incorrect responses such as "38" and "17" were difficult to interpret. This was a very difficult item for the LNP group. Only one student responded correctly in both administrations. Four students responded incorrectly in 1981 but correctly in 1982; two responded incorrectly both years; one responded incorrectly in 1981 and did not attempt in 1982; three did not attempt in 1981 but were correct in 1982; and three did not attempt either time. The most common incorrect responses were "3117", "3107", "318", "307", where incorrect carrying was involved. The incorrect response "415" was evidently a result of some combination of adding and subtracting. Other incorrect responses such as "21", "68", "114" and "9201" are difficult to interpret.

Table 29

Summary of Individual Responses to Item #12, 1981/82 Administrations



Item #12 was responded to correctly both years by five students in the HNP group. Eleven students responded incorrectly in 1981 but correctly in 1982; one

responded correctly in 1981 but incorrectly in 1982; and three responded incorrectly both years. The most common incorrect responses were "366", "306", and "236", where students were evidently not borrowing or borrowing incorrectly. The other most common incorrect responses were "690" and "590" where students were evidently performing the wrong operation. In the INP group, only one student responded correctly in both administrations; seven responded incorrectly in both administrations. Seven responded incorrectly in 1981 but correctly in 1982; one responded incorrectly in 1981 and did not attempt in 1982; one did not attempt in 1981 and responded incorrectly in 1982; and three did not respond either time. The common incorrect responses were "366", "306" where students failed to borrow, or did so incorrectly. Incorrect responses "590", "566", "560" or "360" evidently involved total or parital applications of the incorrect operation of addition. No student in the LNP group had this task correct in both administrations; three responded incorrectly in both administrations; three did not attempt in either administration. Two responded incorrectly in 1981 but correctly in 1982; one responded incorrectly in 1981 and did not attempt in 1982; one did not attempt in 1981 but was correct in 1982; and ten did not attempt in 1981 and responded incorrectly in 1982. Again, the most common incorrect response was "366". The incorrect response "247" evidently involved a carrying idea from tens to ones rather than borrowing. Incorrect responses such as "581" and "6810" involved the use of addition rather than subtraction. Incorrect responses such as "80", "243" and "113" were difficult to interpret.

Following is a set of composite matrices showing percentage of students in each cell based on all twelve items in the interview. A number of observations can be noted. Cell A, the percentage of students in each group responding correctly both years, corresponds to the groups' number performance on entry to first grade.

Eighty percent of the students in the HNP group responded correctly to the twelve items both years, a very high performance rate. Only thirty-eight percent of the students in the LNP group responded correctly to the twelve items both years. Although a relatively low performance rate, it still is noteworthy that a significant percentage of this group with low number ability on entry to first grade did respond correctly to the items during the two administrations. The performance of the INP group remained between the HNP and LNP groups on the tasks.

Table 30

Composite of Individual Responses on Twelve Computation Tasks, 1981/82 Administrations

HNP

1981 \ 1982	C	INC	NA
C	A 80%	B 3%	C 0%
INC	D 15%	E 2%	F 0%
NA	G 0%	H 0%	I 0%

INP

1981 \ 1982	C	INC	NA
C	A 60%	B 7%	C 0%
INC	D 20%	E 6%	F 0%
NA	G 3%	H 2%	I 2%

LNP

1981 \ 1982	C	INC	NA
C	A 38%	B 7%	C 0%
INC	D 20%	E 10%	F 3%
NA	G 8%	H 11%	I 5%

As would be expected, performance of the groups on the other two cells of the diagonal, E and I, were inversely related to performance on entry to first grade ... E reflected the percentage getting items incorrect across both administrations and I the percentage not attempting the tasks across both administrations.

Cells D and G reflected improvement across the two administrations. D indicated those incorrect in 1981 but correct in 1982, and G those not attempting in 1981 but correct in 1982. It is quite noteworthy that in the HNP group, only twenty percent (100% - 80%) improvement was possible and fifteen percent improvement occurred.

On the other hand, in the LNP group sixty two percent (100% - 38%) improvement was possible and only twenty eight percent improvement occurred on the items. In the INP group, forty percent (100% - 60%) improvement was possible, and twenty three percent improvement occurred. The improvement rate with the HNP group was quite high compared to the other two groups. The improvement rate with the LNP group was the lowest of the three groups.

The B and C cells reflected what could be interpreted as inconsistent, or random error performance, on the tasks across the two administrations. The B cell reflects the percentage responding correctly in 1981 but incorrectly in 1982; the D cell reflects the percentage of students responding correctly in 1981 but not attempting to respond in 1982. Relatively few HNP students, 3%, reflected this inconsistent or random error performance, while both the INP and LNP groups reflected a seven percent rate of performance in these cells.

The H and F cells reflect the percentage of students who did not respond in 1981 and then responded incorrectly to the item in 1982, and the percentage of students who responded incorrectly in 1981 and then did not respond to the item in 1982. The LNP group had quite high rates of students represented in these cells compared to the INP group. The HNP group had no students in these cells.

The error analysis carried out on the twelve computation tasks provided some further insight into differences among the three groups of students. Many of the errors made by students were comprehensible to the person examining the responses. Some of the most common errors included: responding with a number adjacent ($\pm$ 1) to the correct response, performing the incorrect operation (adding when they were supposed to subtract, or vice versa) either totally or partially in a problem. Other errors were comprehensible, but unique to the particular task. For example, when three addends were to be processed, it was obvious that some students only

processed two of them and recorded that sum. Other unique, but comprehensible responses were indicated.

In general, the errors made by the HNP group were comprehensible and interpretable by the observer. Their errors were understandable. On the other hand, the LNP group tended to generate more responses to tasks that were incomprehensible on face. On analyzing a number of their responses, it was difficult to interpret how the student arrived at that particular response. The responses of the INP group tended to include somewhat more responses difficult to interpret than the HNP group, but fewer than the LNP group.

Qualitative Analysis --Unprovoked Responses

During both the 1981 and 1982 administrations of the twelve computation tasks, the examiners observed the overt behavior of the students while they were responding to the twelve computation tasks. Three broad categories of responses were observed.

1. No overt manifestations of processing were observed other than those needed to record answers with the pencil.

2. All responses made to the tasks included overt manifestations of processing. These various manifestations included counting on fingers, use of the concrete materials (Recall that ten-rods and unit-cubes were available for student use.), helping marks drawn on paper with accompanying counting, counting while tapping pencil on the paper, or parts of the body.

3. A combination of these two general categories of responses was exhibited.

Table 31 presents the frequency of observations of the three manifestations of processing while addressing the twelve computation tasks.

Table 31

Frequency of Observed Processing Manifestations of Twelve Computation Tasks

		No Overt Manifestations During Processing	Combination of Overt/No Overt Manifestations During Processing	Exclusive Use Of Overt Manifestations During Processing
HNP	1981	60%	35%	5%
Group	1982	65%	20%	15%
INP	1981	35%	35%	30%
Group	1982	20%	45%	35%
LNP	1981	5%	40%	55%
Group	1982	10%	35%	55%

There were quite marked differences among the three groups in manifestations of processing observed. Students in the HNP group typically appeared to use their knowledge of the addition and subtraction combinations in responding to the twelve computation tasks. There appeared to be no dramatic shifts in processing procedures between the 1981 and 1982 observations. On the other hand, very few students in the LNP group appeared to respond with no overt manifestations during processing. Most students in this group were observed using overt manifestations of processing while responding to all, or part, of the twelve computation tasks. There appeared to be little difference in processing procedure between the 1981 and 1982 administrations. The observed processing performance of the INP group was quite evenly distributed among the three categories. Again, there appeared to be little difference in performance between the 1981 and 1982 administrations of the tasks.

Qualitative Analysis--Provoked Responses

When students being interviewed completed the twelve computation tasks, or as many as they could complete, they were presented with a set of unit-cubes and ten-rods that had been available while they were responding the the twelve computation tasks. Each student was allowed to manipulate the materials for a short time and if there was an indication s(he) was unaware of the relationship between the units and tens pieces, the examiner discussed the material and pointed out the relationship between the pieces. The students were then asked to show how they used, or would use, the material to solve two of the computation tasks: 15 - 8, and 23 . The strategies observed being employed by the students from all three
+14
groups (60 observations) during the two administrations (1981 and 1982) are listed below.

Strategies observed in the 15 - 8 task:

Strategy 1: The student made a set of 15 unit cubes, removed 8, indicated the remaining 7 was the answer.

Strategy 2: The student made a linear display of 15 unit cubes and used them to count back from fifteen, stopped after 8 cubes had been touched; the seven untouched cubes in the display were indicated as the answer.

Strategy 3: The students made two parallel rows of unit-cubes, one with fifteen and one with eight. A comparison was made between rows, with the "leftover" in the row of fifteen cubes indicated as the answer.

Strategy 4: The student made a set of 8 and a set of 7 in composing 15. The set of 8 was removed, and the remaining set of seven was indicated as the answer.

Strategy 5: The student initially made a set of 15 composed of a ten-rod and five unit-cubes. After several attempts at a solution, the set was replaced by a set composed of 15 unit-cubes, and then Strategy 1 was employed.

Strategy 6: The student made a set of 15 unit-cubes and a set of 8 unit-cubes. The sets were joined and the total set indicated as the answer. (Incorrect solution)

Strategy 7: The student made a set of 15 with one ten-rod and five unit-cubes. S(he) then systemmatically replaced the ten-rod with ten unit-cubes. Strategy 1 was then employed.

Strategy 8: The student made a set of 15 with one ten-rod and five unit-cubes. S(he) then systemmatically replaced the ten-rod with ten unit-cubes. Two unit cubes were then taken from the set of ten and placed with the set of five, the resulting seven was indicated as the answer.

Strategy 9: The student made two parallel linear displays; one was composed of a ten-rod and five unit-cubes, the second of eight unit-cubes. The student said eight could not be taken from the 5, so s(he) then replaced the ten-rod with ten unit-cubes. Eight unit-cubes were then removed and the remaining set of seven indicated as the answer.

Strategy 10: The student made a set of fifteen composed of a ten-rod and five unit-cubes. The five unit-cubes were removed from the set. One of these was used to determine the distance along the ten-rod which three units would extend. The student then used the width of his/her finger in an iterative counting process to determine how many unit-cubes would be needed to cover the remaining distance on the ten-rod. This count of seven was indicated as the answer.

Strategy 11: The student made a set of fifteen composed of a ten-rod and five unit-cubes. S(he) then arranged eight unit cubes along the side of the ten-rod and said eight and two was ten, and the two (complement of eight) plus five unit-cubes was seven--the answer.

Strategy 12: The student made a set of fifteen composed of a ten-rod and five unit-cubes. S(he) then removed the five unit-cubes and counted to three while touching a section of the ten-rod. The student then indicated the remaining part of the ten-rod was seven--the answer.

Strategy 13: The student made two parallel linear displays, first a set of fifteen composed of one ten-rod and five unit-cubes, then below it a set composed of eight unit-cubes. The set of eight unit-cubes was removed and the remaining set indicated as the answer. (Incorrect solution)

Strategy 14: The student made a set composed of one ten-rod and five unit-cubes. The ten-rod was removed and replaced by five unit-cubes, resulting in a display of 10 unit-cubes. Eight unit-cubes were then removed from the set. (Incorrect solution)

Strategy 15: The student made three parallel linear displays, the first of fifteen unit-cubes, the second eight unit-cubes, and the third seven unit-cubes. S(he) then pointed out that fifteen take away eight was seven.

Strategy 16: The student made a set of fifteen unit-cubes. S(he) took eight away from the set, and then took five from the remainder, leaving three, which was indicated as the answer. (Incorrect solution)

Strategy 17: The student made a linear display of fifteen composed of one ten-rod and five unit-cubes. S(he) tehn made another linear display of eight unit-cubes, then simply said the answer was seven, but could not show how it was arrived at.

Strategy 18: The student made a set of fifteen, composed of one ten-rod and five unit-cubes. Three more unit-cubes were then placed with the five already there. The answer was indicated as ten. (Incorrect solution)

Strategy 19: The student made a set of fifteen unit-cubes, and then a set of five unit-cubes. The answer was indicated as twenty. (Incorrect solution)

Table 32

Percent of Students in Each Group Using Particular Strategies During 1981 and 1982 Test Administrations of 15 - 8

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
HNP 1981	85%	0%	5%	5%	5%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Group 1982	25%	0%	0%	0%	0%	0%	45%	5%	5%	5%	5%	5%	0%	5%	0%	0%	0%	0%	0%
INP 1981	90%	5%	0%	0%	0%	5%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Group 1982	70%	0%	0%	5%	0%	0%	0%	0%	0%	0%	0%	0%	5%	0%	5%	5%	5%	5%	0%
LNP 1981	90%	5%	0%	0%	0%	5%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Group 1982	80%	0%	0%	5%	0%	5%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	5%	5%

The most dramatic shift between the 1981 and 1982 administrations occurred with the HNP group. Most of the students used Strategy #1 during the 1981

administration. This involved interpreting "15" as fifteen unit-cubes and then removing eight unit-cubes from that set. A year later the majority of students in the HNP group had shifted to strategy #7. This involved interpreting "15" as one ten-rod and five unit-cubes. Strategies 8, 9, 10, 11 and 12 used by students in the HNP group also involved interpreting "15" as one ten-rod and five units. Evidently the numeral driven interpretation of "15" as one ten and five units and the decomposition of the ten-rod to ten-units in the demonstration of the subtraction process was quite well developed among the HNP group at the time of the second administration.

On the other hand, both the INP group and the LNP group tended to use Strategy #1 both years. There was little evidence in their demonstrations that they were interpreting the "15" as composed of one ten and five units.

It is of interest to point out that 85%, 70% and 65% of the HNP, INP and LNP groups respectively responded to this item (#4) correctly in 1981; 95%, 80%, and 85% of the HNP, INP and LNP groups respectively responded correctly in 1982. On face it would appear that there was much similarity in the ability to respond correctly to this task among students in the three groups. However, when provoked to demonstrate their processing with tens and ones materials, there appeared to be quite a dramatic difference between the HNP group and the other two groups. The HNP group appeared to have a high frequency of students interpreting fifteen as composed of ten as one unit and five discrete units by the time of the 1982 administration. Few students in either of the other two groups appeared to be interpreting fifteen in that way; they tended to demonstrate fifteen as that many discrete objects.

Strategies observed in the 23 task:

$$\underline{+14}$$

Strategy 1: The student made two sets. The first was composed of two ten-rods and three unit-cubes; the second was composed of a

ten-rod and four unit-cubes. The sets were placed either side-by-side vertically or one beneath the other horizontally. The student then put together the ten-rods, then the unit-cubes, in demonstrating the answer 37.

Strategy 2: The student made sets as described in Strategy #1, but put together the unit-cubes first and then the ten-rods.

Strategy 3: The student made a set of three unit-cubes and a set of four unit-cubes, and then put them together. S(he) then made a set of one ten-rod and a set of 2 ten-rods, and then put them together.

Strategy 4: The student made two sets. The first was composed of two ten-rods and three unit-cubes; the second was composed of fourteen unit-cubes. The sum was determined by counting the elements in the two sets by the two tens and seventeen ones, e.g., ten, twenty, twenty-one ... thirty-seven.

Strategy 5: The student made two sets. The first was composed of twenty three unit-cubes; the second was composed of one ten-rod and four unit-cubes. The counting procedure using the ten-rod and then unit counting as in Strategy #4 was then carried out.

Strategy 6: The student made a set composed of twenty three unit-cubes, and another composed of fourteen unit cubes. The sum was then determined by counting the cubes by one in the combined set. (In some cases ten-rods were used as unit-cubes in composing the sets. Apparently the rods were perceived as a single element in the set to be counted the same as the unit-cubes.)

Strategy 7: The student made a set composed of twenty three unit-cubes. S(he) then removed fourteen of the unit cubes. (Incorrect solution)

Strategy 8: (The student used either the ten-rods or the unit-cubes interchangeably in this strategy. Both were evidently perceived as objects to be counted as single units; difference in dimensions appeared to be ignored.) The student formed a set of one, then a set of two, then joined them and counted them -- one, two, three. A set of three, then a set of four, was then joined and counted -- one, two, ... seven. It It appeared that the student was perceiving two separate addition combinations: 2 + 1 and 3 + 4.

Strategy 9: The student would say "three", and then count on "four", "five", "six", "seven" while touching four unit-cubes. S(he) would then say "one" and then count on "two", "three" while touching two unit-cubes. Again, it appeared that the student was perceiving two separate addition combinations: 2 + 1 and 3 + 4.

Strategy 10: The student made two sets. The first was composed of one ten-rod and thirteen unit-cubes; the second was composed of one ten-rod and four unit-cubes. The ten-rods and the unit-cubes were joined separately. Then ten unit-cubes were put with the set of two ten-rods to demonstrate the answer -- three tens and seven ones, 37.

Table 33 indicates the use of these strategies by students in the three groups in the 1981 and 1982 administrations. It can be noted that there were some marked differences among students in the three groups in the strategies used to demonstrate their processing.

Table 33

Percent of Students in Each Group Using Particular Strategies During 1981 and 1982 Administrations of $\frac{23}{+14}$

		1	2	3	4	5	6	7	8	9	10	No Response
HNP	1981	30%	25%	10%	10%	0%	0%	15%	0%	0%	0%	10%
Group	1982	55%	10%	0%	20%	10%	0%	5%	0%	0%	0%	0%
INP	1981	5%	15%	0%	25%	0%	5%	15%	0%	15%	5%	20%
Group	1982	30%	0%	5%	25%	10%	0%	15%	0%	10%	0%	5%
LNP	1981	5%	5%	0%	25%	0%	0%	5%	10%	25%	5%	20%
Group	1982	0%	0%	10%	25%	15%	5%	20%	0%	10%	0%	15%

There appeared to be a quite high frequency in use of strategies #1 and #2 by students in the HNP group. Few students in the LNP group used those strategies. The frequency in use of the strategies by the INP group fell between these extremes. It would appear that the use of strategies #1 and #2 corresponded with students' number abilities on entry to first grade. The strategies could be characterized as quite mature use of tens and ones materials in demonstrating the addition task.

Strategy #4 was another quite high frequency strategy. It was used consistently by twenty five percent of the INP and LNP groups across both administrations. It was also used by the HNP group, but with somewhat less frequency. The strategy could be characterized as being somewhat less mature in the use of tens and ones materials, since the use of the ten-rods as a unit composing a number was only applied in one of the addends. The other addend was demonstrated as discrete units.

Strategy #7 demonstrated the task, incorrectly, as a subtraction situation. There was a decrease in use of this strategy between 1981 and 1982 by the HNP group, an increase by the LNP group, and a constant use by the INP group. In addition to demonstrating an incorrect operation, it did also not involve the use of any ten-rods in the composition of the numbers.

Strategy #9 was used by some students in both the INP and LNP groups. No students in the HNP group used it. It involved no use of tens materials and it appeared as if the students were demonstrating two distinct additions: $\begin{array}{r} 2 \\ +1 \end{array}$ and $\begin{array}{r} 3 \\ +4 \end{array}$.

It is again interesting to note that all three groups were responding correctly to this task on the paper/pencil presentation at quite a high rate of frequency. Ninety five percent, 85%, and 80% respectively of the HNP, INP, and LNP groups responded correctly to this item on the 1982 administration. On face it might appear that students in all three groups have a similar performance level on this task. However, when asked to demonstrate their solution with tens and ones materials, there are marked differences in their performance. The great majority of the HNP group use the ten-rods in composing numbers in demonstrations. Fewer students in the INP group use ten-rods in their demonstrations, and relatively few students in the LNP group use tens-rods in number compositions to use in the demonstrations.

School Arithmetic Development: Word Problems

The students in each group of the study were administered four word problems during their individual interviews in 1981 and 1982. Each student was given a sheet of paper with the four problems printed on it. The examiner read each question to the student who was given time to respond before the next problem was read. All the problem types were classified as part-part-whole (Carpenter and Moser, 1979). Two were addition problems; two were subtraction problems. One of the addition and one of the subtraction problems contained extraneous information. The four word problems are shown in Figure 4.

Quantitative Analysis: The mean and standard deviation of performance scores for the HNP, INP and LNP groups for both the 1981 and 1982 administrations are shown in Table 34. It is quite apparent that the HNP group had little difficulty with these tasks in either the 1981 or 1982 administrations. Their performance was significantly better on the tasks than either the INP or the LNP groups. The performance of these groups on the four word problems was very similar in 1981. In 1982 the INP group showed some improvement, while the LNP group performance was not as good as it had been in 1981.

Table 34

Statistical Data on Word Problem Performance of the Three Groups

	Spring 1981		Spring 1982	
	$\bar{X}$	Sd	$\bar{X}$	Sd
HNP Group	3.5	0.72	3.65	0.49
INP Group	1.9	1.37	2.20	1.11
LNP Group	1.8	1.44	1.35	1.31

NAME_____

Sue has 9 balloons. 6 are red and the rest are blue.
How many of the balloons are blue?

Dan has some gumdrops. 8 are mint and 4 are lemon.
How many gumdrops does he have?

Altogether there were 16 runners who started a race.
7 of them finished the race. It took them 8 minutes.
How many runners did not finish the race?

Altogether there were 8 players with blue shirts and 9 with white shirts on the football field. They had 6 footballs.
How many players were on the field?

Figure 4. Word Problems Used in 1981/82 Administrations

Table 35 shows the performance of students in the three groups on each of the four word problems. Overall the two subtraction word problems were more difficult than the two addition problems. Within the set of two addition or subtraction problems the one with extraneous information was more difficult than the one without. The most difficult problem was the subtraction with extraneous information; the easiest was the addition problem without extraneous information. These observations were generally true across all groups.

Table 35

Percent of Students Responding Correctly to Word Problems

Administration		1 Subtract	2 Add	3 Subtract Ext.	4 Add Ext.
HNP	1981	80%	95%	65%	85%
Group	1982	100%	100%	75%	90%
INP	1981	45%	60%	40%	45%
Group	1982	55%	70%	35%	55%
LNP	1981	45%	60%	30%	45%
Group	1982	20%	55%	10%	50%

The HNP group showed improvement in performance on each of the four problems, even though their initial performance was high and the ceiling on the test restricted any dramatic gains. As with their performance on the twelve computation tasks, they had a highly restricted improvement range between the 1981 and 1982 administrations, but did show improvement within that restricted range.

The INP group showed improvement on three of the four problems between the 1981 and 1982 administrations. There was a very slight regression in performance

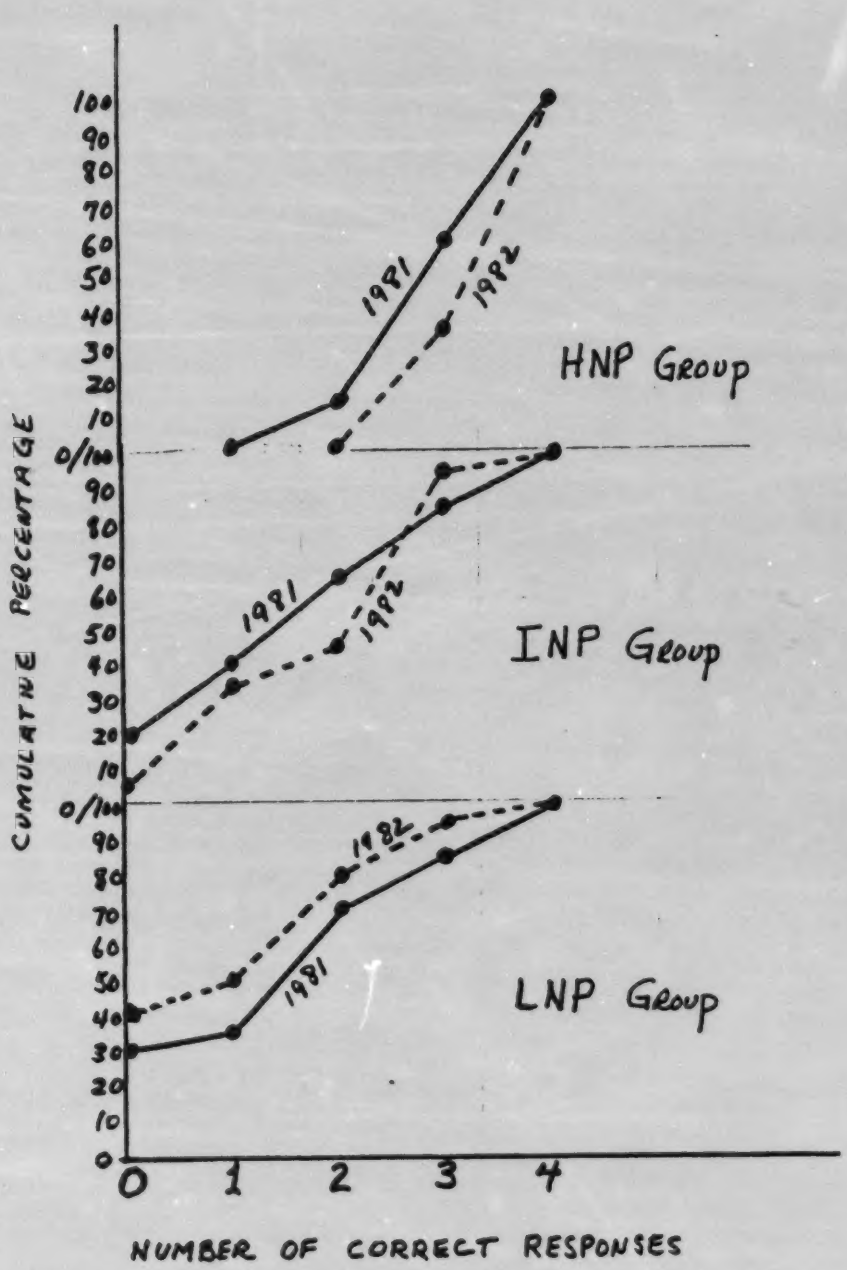


Figure 5. Ogives of Performance on Four Word Problems

on the subtraction problem with extraneous information.

The LNP group showed only slight improvement on one task, the addition with extraneous information. Their performance on the two subtraction problems showed quite a marked deterioration between the 1981 and 1982 administrations. Although the LNP group's performance on the word problem tasks were very similar to the INP group's performance at the time of the 1981 administration, there was quite marked differences in 1982 between the groups as a result of the apparent progress of the INP group, and apparent regressive performance of the LNP group.

The ogives shown in Figure 5. provide further evidence of the similarities and differences of the three groups on the four word problems. It can be noted that at the time of the 1981 administration, only students at or above the median in performance in both the INP and LNP groups were within the range of scores of the HNP group. Those below the median were below the entire range of scores of the HNP group. This generally remained true for both the LNP and INP groups at the time of the 1982 administration.

Qualitative Analyses of Word Problems:

After the student's initial response to each word problem, the student was asked why s(he) responded as s(he) did. Following is a list of strategies used in getting solutions to the four word problems. Strategies leading to correct responses:

1. Used an appropriate addition combination at the symbolic level.
2. Used an appropriate addition combination in conjunction with concrete materials.
3. Used an appropriate subtraction combination at the symbolic level.
4. Used an appropriate subtraction combination in conjunction with concrete materials.
5. Used a counting procedure to find the solution.
6. Used a partitioning procedure in separating a set into subsets.
7. Counted on from one addend.
8. Counted back from a given number.

Strategies leading to incorrect responses:

9. Used an addition combination at the symbolic level, but selected incorrect numbers or added incorrectly.
10. Used a subtraction combination at the symbolic level but selected incorrect numbers or subtracted incorrectly.
11. Appeared to simply scan the problem statement and responded with a number given in the problem.
12. Counted or used the concrete materials, but arrived at incorrect solution.
13. Could give no response.
14. Other.

Table 35 shows the strategies used by students in responding to word problem #1.

Table 35

Percent of Students Using Strategies in Responding to First Word Problem

		CORRECT STRATEGIES								INCORRECT STRATEGIES					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
HNP	1981	0	10%	65%	0	0	0	0	0	10%	0	10%	0	5%	0
Group	1982	35%	0	60%	0	0	0	0	0	5%	0	0	0	0	0
INP	1981	20%	0	15%	0	0	10%	5%	0	5%	0	20%	0	15%	10%
Group	1982	25%	0	35%	0	0	0	0	0	20%	0	20%	0	0	0
LNP	1981	0	0	35%	0	0	0	0	5%	15%	0	40%	0	5%	0
Group	1982	5%	0	15%	0	0	0	0	0	40%	0	30%	10%	0	0

The HNP group typically used the appropriate subtraction combination, 9 - 6= 3, at the symbolic level as a strategy in responding to the problem situation. There was an increase in the tendency to use the appropriate addition combination 6 + 3=9 at the time of the 1982 administration.

Those students in the INP group responding correctly to the problem used either the appropriate addition or subtraction combination at the symbolic level at the time of the 1982 administration. A few students, at the time of the 1981 administration made two appropriate sets with the manipulative materials to find the solution, and one student used a counting-on procedure. These less mature strategies were not evident by the time of the 1982 administration. Those students who got the problem incorrect generally tended to use an addition procedure but added incorrectly, or just responded with a number imbedded in the word problem, indicating that they were not really cognitively involved in the problem (Strategy #11).

Those students in the LNP group who responded correctly tended to use either

the appropriate addition or subtraction combinations at the symbolic level. There seemed to be rather a strong tendency for students to use the addition procedure at the time of the second administration, and then add incorrectly. There was quite a high frequency of students in this group who used strategy #11, which simply involved responding with one of the numbers they perceived imbedded in the problem narrative.

Table 36

Percent of Students Using Strategies in Responding to Third Word Problem

		CORRECT STRATEGIES								INCORRECT STRATEGIES					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
HNP	1981	0	0	60%	5%	0	0	0	0	10%	25%	0	0	0	0
Group	1982	15%	0	60%	0	0	0	0	0	0	25%	0	0	0	0
INP	1981	10%	0	30%	5%	0	0	0	0	10%	15%	15%	0	10%	5%
Group	1982	5%	0	40%	0	0	0	0	0	15%	15%	15%	10%	0	0
LNP	1981	0	0	15%	20%	0	0	0	0	15%	15%	30%	0	0	5%
Group	1982	0	0	15%	5%	0	0	0	0	25%	20%	35%	0	0	0

Table 36 shows the strategies used by students in responding to word problem #3. Again, the HNP group typically used the appropriate subtraction combination, 16 - 7= 9, at the symbolic level in responding to the problem. There was also a slight tendency for some students to be using the appropriate addition combination, 7 + 9 = 16, by the time of the second administration. Those responding incorrectly tended to use the subtraction combination, but processed it incorrectly or chose the incorrect numbers.

Those in the INP group who responded correctly tended to use either the

appropriate addition or subtraction combination. Those responding incorrectly generally distributed their strategies across all the different incorrect procedures observed.

Most of the students in the LNP group responded incorrectly. Many of them simply responded with a number they perceived in the problem narrative. Many others used a subtraction operation but either processed it incorrectly or chose the incorrect numbers. Those few in this group who responded correctly tended to use the appropriate subtration combination at the symbolic level or with concrete materials as an aid in finding the solution.

Table 37 shows the strategies used by students in responding to word problem #2. Essentially all students in the HNP group responded correctly on both administrations and used the appropriate addition combination at the symbolic level, 8 + 4 = 12. About three-fourths of the INP group responded correctly in both administrations and used the addition combination at the symbolic level, but there were a few in this group who used a counting-on procedure at the time of the first administration. Those responding incorrectly either chose the incorrect addition operation, processed the subtraction incorrectly, or just chose one of the numbers in the narrative and gave it as the answer. Those in the LNP group responding correctly used the appropriate addition combination, either at the sumbolic level or with the aid of concrete materials by the 1982 administration. There were some at the time of the 1981 administration who used a counting procedure to get the correct answer. Of those responding incorrectly, many, again, responded simply with a number from the problem. Other incorrect responses were distributed across various incorrect strategies.

Table 38 shows the strategies used by students in responding to word problem #4. Most of the HNP group responded correctly and used the appropriate addition combination, 8 + 9 = 17, at the symbolic level. Those few responding incorrectly either chose the incorrect operation, or incorrect numbers to use in the addition.

Table 37

Percent of Students Using Strategies in Responding to Second Word Problem

		CORRECT STRATEGIES								INCORRECT STRATEGIES					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
HNP	1981	95%	0	0	0	0	0	0	0	0	5%	0	0	0	0
Group	1982	100%	0	0	0	0	0	0	0	0	0	0	0	0	0
INP	1981	50%	0	0	0	0	0	15%	0	10%	15%	10%	0	0	0
Group	1982	75%	0	0	0	0	0	0	0	5%	15%	5%	0	0	0
LNP	1981	25%	25%	5%	0	15%	0	0	0	0	0	20%	0	0	10%
Group	1982	40%	15%	0	0	0	0	0	0	5%	10%	15%	10%	0	5%

Table 38

Percent of Students Using Strategies in Responding to Fourth Word Problem

		CORRECT STRATEGIES								INCORRECT STRATEGIES					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
HNP	1981	80%	5%	0	0	0	0	0	0	10%	5%	0	0	0	0
Group	1982	90%	0	0	0	0	0	0	0	0	10%	0	0	0	0
INP	1981	45%	5%	0	0	0	0	0	0	30%	5%	10%	0	5%	0
Group	1982	55%	20%	0	0	0	0	0	0	25%	10%	5%	0	0	0
LNP	1981	25%	20%	0	0	0	0	0	0	15%	5%	20%	0	10%	5%
Group	1982	40%	5%	0	0	0	0	0	0	20%	0	25%	0	10%	0

In the INP group about 75% were responding correctly by the 1982 administration; they were using the appropriate addition combination, either at the symbolic level or with the aid of concrete materials. Most of the incorrect responses were the result of selecting incorrect numbers for the addition. A little less than 50% of the LNP group responded correctly to this problem. All used the appropriate

addition combination, either at the symbolic level or with the aid of concrete materials. Those responding incorrectly tended to use the wrong numbers in the addition or, again, just chose a number from the problem narrative as the response.

In general, the HNP group was quite markedly superior to the INP and the LNP groups on the four word problems on both the 1981 and 1982 administrations of the tasks. They were generally able to choose the correct operation for the problems and then apply the appropriate number combination and process the task at the symbolic level. Few students in the HNP group were affected by the extraneous information in a problem. The tasks were more difficult for the INP group, but there was growth in the group's performance between the 1981 and 1982 administrations. Those who were responding correctly were choosing the correct operation and the correct number combination, then processing at the symbolic level. The LNP group did very poorly on these tasks on both administrations. Their performance during the 1982 administration was generally worse than at the time of the 1981 administration. Generally, there was more frequency of students who would choose the incorrect operation, be distracted by the extraneous information in a problem, or would use a simplistic response such as choosing a number from the problem narrative for the response. In spite of the fact that only numbers that fell within the range of the basic addition and subtraction combinations were used in the problems, the LNP group was performing very poorly on these tasks after three years of school.

4. School Arithmetic Development: Commutative and Identity Properties of Operations

Following the students' completion of the twelve computation tasks and the four word problems, they were presented with activities that involved them in responding to questions about commutative and identity properties. To focus attention on the task, the interviewer used a simple "function machine" made from a milk carton. A card with a number combination on one side and the answer on the other side was inserted in a slot at the top of the carton. When inserted, the combination side was visible to the student; after passing through the carton, the card would emerge at the bottom with the answer visible. There were two machines, one for addition and one for subtraction.

Commutative Property: Students were given two illustrations of how the machine operated. They were then shown a display board with two related combinations as shown at right. To assure the students were attending to the difference/similarities of the two combinations, they were each asked a sequence of three questions.

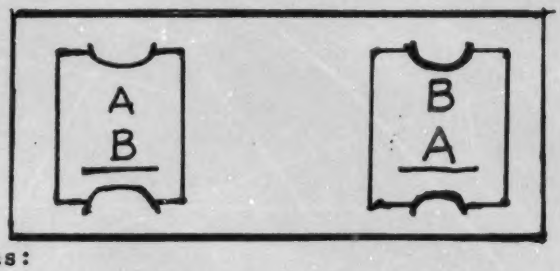


Figure 6. Combination Display Boards

1. What is alike about these two combinations:
2. What is different about them?
3. Which "machine" should be used?

The card on the left would then be removed from the display board and the student asked, "When I put this card through the machine, what will be the answer that will come out the bottom?" The card would then be sent through the machine to check the student response, or to provide the answer if the student was unable to give the answer. Then, without removing the card on the right from the display board, the interviewer would then ask the student, "What if I put this card through the machine? What would be the answer that would come out the bottom? Why?"

This procedure was used with four display boards, one set of simple addition combinations, one set of more difficult addition combinations, one set of simple subtraction combinations, one set of more difficult subtraction combinations. The four displays are shown below.

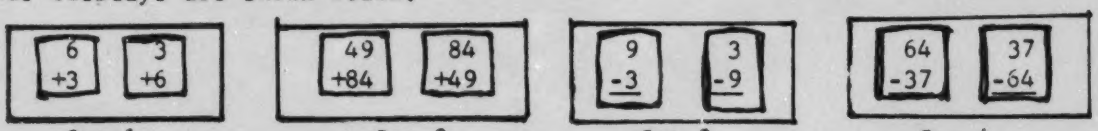


Figure 7. Four Display Boards Used in Presenting Tasks

The criterion for judging students' knowledge of commutativity as it applied to the operation of addition was their responses to the "What if ..." question about the combination on the right of the display board of set 1 and 2.

Table 40 presents information about student responses to the two sets of addition tasks for 1981 and 1982. A correct response was one where the student indicated that the answer to the combination on the right of the board would be the same as the answer provided by the machine to the combination on the left that had been sent through the machine.

Table 40

Percentage of Correct Responses to Addition Commutativity Questions

		Correct Response, Simple Addition	Correct Response, Difficult Addition
HNP	1981	100%	100%
Group	1982	100%	100%
INP	1981	100%	100%
Group	1982	100%	100%
LNP	1981	95%	85%
Group	1982	100%	100%

Table 41 presents information about student responses to the two sets of subtraction tasks for 1981 and 1982. A correct response was one where the student indicated that the answer to the combination on the right of the board would not be the same as the answer provided by the machine to the combination on the left that had been sent through the machine.

It appeared that students in all groups had quite a stable concept of commutivity as applied to the operation of addition. Even though many students in the groups did not know the answer to the difficult addition combination until it was provided by the machine, yet they indicated the answer to the combination on the right of the display board would be the same.

Table 41

Percentage of Correct Responses to Subtraction Non-Commutativity Question

		Correct Response, Simple Subtraction	Correct Response, Difficult Subtraction
HNP	1981	50%	40%
Group	1982	35%	30%
INP	1981	25%	15%
Group	1982	20%	10%
LNP	1981	0%	0%
Group	1982	15%	15%

The responses to the subtraction questions were quite different. Although there were more students in the HNP group indicating that the answer to the combination on the right of the display board would not be the same as the answer provided

by the machine to the subtraction on the left, most of the students in all groups indicated the answer would be the same. Most students in the INP and LNP groups apparently feel that the order of the numbers in a subtraction situation will not affect the answer. There appeared to be no increase in correct responses for the HNP and INP group between the two administrations; in fact there was some decrease in frequency of correct responses by students in these groups.

In response to the "Why" question, after responding to the set of addition combinations, four categories of responses were observed.

1. Students focused only on the addends being the same, e.g., "Because it's the same numbers as over here."

2. Students focused on the addends being the same and the order changed, e.g., "It's the same problem except it's turned around."

3. Students focused on the sum, or result, of the operation, e.g., "Because they both got 9." or "Because that (Pointed to combination on left side of display board) was 133."

4. Students could give no reason.

Table 42 indicates the frequency of these rationales given by the three groups of students when asked "Why" they gave the response to the "What if ..." question asked about the addition combinations on the right of the display boards.

Table 42

Percentage of Students Using Various Rationales in Addition

		Rationales							
		1		2		3		4	
		Simple +	Difficult +	Simple +	Difficult +	Simple +	Difficult +	Simple +	Diff. +
HNP	1981	70%	65%	25%	20%	0%	5%	5%	10%
Group	1982	50%	60%	45%	35%	5%	5%	0%	0%
INP	1981	75%	60%	15%	30%	10%	10%	0%	0%
Group	1982	70%	65%	20%	25%	10%	10%	0%	0%
LNP	1981	45%	70%	20%	10%	20%	0%	15%	20%
Group	1982	55%	50%	20%	40%	25%	10%	0%	0%

There appeared to be little difference among the groups on the rationales used to defend their responses to the addition tasks. Most indicated they responded the way they did because the numbers were the same as those in the companion combination, and only the order was different.

In response to the "Why" question after responding to the set of subtraction combinations, five categories of responses were observed.

1. Students focused on the subtrahend being greater than minuend, e.g., "Because 3 canct take away 9." or "Nine is bigger than 3, so you won't get an answer."

2. Students focused on the numbers in the problem being the same, e.g., "Because they are the same numbers."

3. Students focused on the numbers in the problem being the same and the order being different, e.g., "They're the same numbers, just turned around."

4. Students focused on the answer, or the result of the operation, e.g., "Because I saw that one (pointing to the combination on the left of the display board) was 27."

5. Students could give no reason.

Table 43 indicates the frequency of these rationales given by the three groups of students when asked "Why" they gave the response to the "What if ..." question asked about the subtraction combinations on the right of the display boards.

Table 43

Percentage of Students Using Various Rationales in Subtraction

		Rationales									
		1		2		3		4		5	
		Simple	Diff.	Simple	Diff.	Simple	Diff.	Simple	Diff.	Simple	Diff.
HNP	1981	30%	30%	35%	30%	20%	25%	0%	10%	15%	5%
Group	1982	25%	25%	35%	60%	30%	15%	10%	0%	0%	0%
INP	1981	20%	15%	50%	55%	20%	10%	5%	5%	5%	15%
Group	1982	15%	10%	50%	60%	15%	15%	15%	10%	5%	5%
LNP	1981	0%	0%	50%	45%	20%	20%	0%	0%	30%	35%
Group	1982	10%	5%	60%	50%	15%	25%	10%	10%	5%	10%

The first rationale was the only one associated with a correct response, and it can be noted that there were a few more HNP group students responding in that category than INP group students. Few LNP group students responded with that rationale, even at the second administration. Most students in all groups gave a rationale (#2 or #3) that indicated that since the numbers were the same and only the order was different, the answer to the combination would be the same. It would appear that the typical student believes the order in which two numbers are subtracted will not affect the answer.

Identity Property: Four sets of tasks were presented to each student on display boards as shown below:

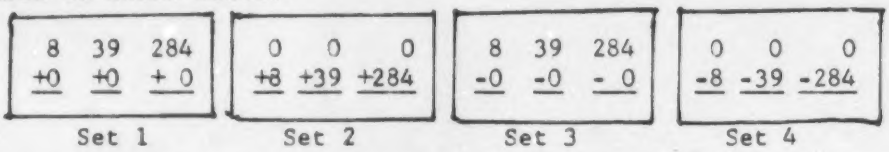


Figure 8. Display Boards Used in Identity Property Tasks

For each set of combinations the one on the left would be removed from the display board and the question asked, "What would be the answer when I (add/subtract) these two numbers?" The card was then sent through the machine to check the accuracy of the response, or provide the answer if the student was unable to respond. Then the examiner asked, "What would be the answer if I (added/subtracted) these numbers?" (While pointing to the other two combinations on the display board.) In the case of the $\frac{0}{-8}$ card, no answer was placed on the reverse side of the card, so it would come out the bottom of the machine with a blank side displayed. The examiner would ask each student why s(he) thought the machine gave no response to that combination.

Table 44 indicates the frequency of correct responses to the four sets of tasks. The first three sets of tasks were quite easy for all groups. It appeared that the zero generalization for addition was quite stable for all the students.

However, as with the previous set of tasks that focused on the commutative idea, the students again appeared to believe that the order of numbers to be subtracted will not affect the answer, and were therefore incorrect in responding to the set #4 tasks. About one half of the HNP group responded correctly by indicating they could not give an answer to the task. Only a few LNP group students responded correctly.

Table 44

Percentage of Students Responding Correctly to Four Sets of Tasks

		Set #1	Set #2	Set #3	Set #4
HNP	1981	100%	95%	100%	50%
Group	1982	100%	100%	100%	45%
INP	1981	95%	95%	90%	30%
Group	1982	100%	85%	85%	35%
LNP	1981	95%	90%	85%	15%
Group	1982	100%	100%	85%	15%

In general, it appeared that all students could apply the commutative idea and the identity property of zero as they apply to addition. However, there is a strong tendency to overgeneralize the commutative idea to the operation of subtraction. This tendency appeared to be present in all three groups of students, but somewhat more prevalent with the LNP group students than with the HNP students.

5. Arithmetic Program Descriptions

At the end of each school year during the study, teachers were asked to respond to a series of questions about aspects of the mathematics program experienced by the students in the study. As indicated in the introductory section, it was only in the first grade that a teacher would have all three students from a given trio in the same classroom. Following that first-grade experience, students could not be controlled as to classroom setting. This section presents descriptions of the mathematics programs experienced by students in the HNP, INP, and LNP groups druing their first-grade experience and during the ensuing two years of the study. Teachers' perceptions of the maturity levels of the students will also be presented.

The first important consideration in programming is the fact that a significant number of students in the LNP group were retained in the first grade. Twelve of the twenty students in the group repeated first grade; one was placed in a special class. At the completion of the study, there were only seven students in this group who had completed the third grade. The remainder had completed the second grade, indicating that they had only been held back one year. Four students in the INP group repeated first grade; no students in the HNP group were retained. It would appear that poor performance on the number skills tasks on entry to first grade is quite a valid predictor of students at risk as to normal school progress. In the following sections, other descriptions of the arithmetic program are presented.

Classroom Time Spent on Arithmetic Instruction

Teachers were asked to indicate the approximate amount of time spent on arithmetic activities daily within the classroom. They could choose from four time intervals: 0 - 15 minutes, 15 - 30 minutes, 30 - 45 minutes, and 45 - 60 minutes. Table presents the responses to this question for the three years for the three groups.

Table 45

Minutes Per Day of Arithmetic Instruction

	0 - 15	15 - 30	30 - 45	45 - 60
HNP Group	0%	10%	69%	21%
INP Group	0%	17%	57%	26%
LNP Group	0%	17%	57%	25%

It would appear that the amount of time spent in class on arithmetic instruction did not differ greatly among the three groups of students. The typical teacher spent about 30 -45 minutes of classroom instruction time on arithmetic.

Arithmetic Programs Used by Students

Teachers were asked to indicate the name of the program used in the arithmetic experience. Those programs used by students in the LNP group during their three year experience were: Harbrace, Silver Burdett, Scott Foresman, Random House, Houghton Mifflin, STAMM, DMP, and DISTAR. Programs used by students in the INP group during their three year experience were: Silver Burdett, Scott Foresman, Random House, Houghton Mifflin, Holt, DMP, and DISTAR. Programs used by students in the HNP group during their three year experience were: Silver Burdett, Scott Foresman, Random House, DMP and DISTAR.

Table 46 indicates the percentage of students in the three groups who: (a) experienced the same program during all three years covered by the study, (b) experienced the same program two of the three years, (c) experienced different programs all three years of the study.

Table 46
Percentage of Students Experiencing Same/Different Programs

	Three Years in Same Program	Two of Three Years in Same Program	All Different Programs
HNP Group	15%	75%	10%
INP Group	17%	55%	28%
LNP Group	10%	60%	25%

Teachers were also asked to indicate the frequency which they used the program materials, and if they completed the textbook or the designated grade level materials during the year. Table 47 indicates those responses.

Table 47

Program Use Across Three Years for Three Groups

	Frequency of Use			Completed	
	Daily	3/4 Weekly	1/2 Weekly	Yes	No
HNP Group	70%	25%	5%	65%	35%
INP Group	70%	25%	5%	55%	45%
LNP Group	75%	15%	10%	55%	45%

Students in the three groups experienced quite a variety of programs during the three years of the study. Very few students were in the same program for all three years. Typically, the students experienced two of the three years in the same program. There appeared to be a tendency for more students in the INP and LNP groups to experience different programs each year than students in the HNP group. All groups of students typically used the program materials daily, with about all using them at least three or four days a week. The majority of students in all three groups completed the materials at particular grade levels, with a slight tendency for more HNP group students completing the materials during the year than students in the INP and LNP groups.

Supplementary Assistance for Students

Teachers were asked to indicate whether students received supplementary arithmetic experiences outside of classroom instruction. Typically, this would involve a Title I mathematics corrective teacher or resource teacher to work with students in the LNP and INP groups, and resource teachers that would provide enrichment experiences for students in the HNP group. Table 48 indicates the frequency of

supplementary assistance provided during the three years of the study.

Table 48

Percentage of Students Receiving Supplementary Arithmetic Instruction

	1979/80	1980/81	1981/82
HNP Group	30%	30%	25%
INP Group	30%	45%	60%
LNP Group	80%	15%	45%

There was a marked difference between the LNP group and the other two groups in supplementary instruction received during the first grade experience. Most of the LNP students were receiving supplementary assistance, while this was not true for the HNP and INP groups. During the second year of the study, few LNP students were receiving supplementary assistance. This can probably be accounted for by the large number in this group repeating first grade. Many may not have been eligible for assistance. During the third year there were an increasing number of INP and LNP students receiving supplementary instructional assistance in arithmetic. Table 49 indicates the percentage of students in each group receiving 3, 2, 1 and 0 years of supplementary instructional assistance during the duration of the study.

Table 49

Amounts of Supplementary Instruction During Three Years of the Study

	Three Years	Two Years	One Year	0 Years
HNP Group	15%	10%	25%	50%
INP Group	15%	25%	40%	20%
LNP Group	10%	30%	50%	10%

Although most of the supplementary assistance was provided by various title

teachers, there was some provided by parent volunteers, aides, and older student tutors. The amount of time supplementary assistance was provided varied from as much as thirty minutes per day to as little as thirty minutes per week.

Arithmetic Curriculum Emphasis

Teachers were asked to indicate the relative order of emphasis (1, 2, 3 or 4, with 1 being high emphasis and 4 low emphasis) given four categories of arithmetic experience: (a) Number Development Activities (ND), (b) Measurement and Geometry (MG), (c) Addition and Subtraction Facts and Computing (FC), (d) Word Problems using Addition and Subtraction Operations (WP). Tables 50, 51, and 52 indicate the responses of the teachers for the three years of the study.

Table 50

Frequency of Teachers of LNP Group Indicating Topic Emphases

		1980 Priorities				1981 Priorities				1982 Priorities			
		1	2	3	4	1	2	3	4	1	2	3	4
TOPICS	Number Development	70%	15%	10%	5%	45%	35%	5%	15%	20%	45%	35%	0%
	Mea/Geom.	0%	25%	55%	5%	15%	25%	35%	25%	5%	30%	25%	40%
	Facts/Comp.	35%	55%	10%	0%	70%	30%	0%	0%	80%	15%	0%	5%
	Word Prob.	0%	5%	45%	50%	5%	50%	15%	30%	40%	10%	25%	25%

Table 51

Frequency of Teachers of INP Group Indicating Topic Emphases

		1980 Priorities				1981 Priorities				1982 Priorities			
		1	2	3	4	1	2	3	4	1	2	3	4
TOPICS	ND	55%	30%	15%	0%	50%	35%	5%	10%	30%	15%	50%	5%
	MG	5%	25%	35%	35%	10%	25%	35%	30%	0%	30%	25%	45%
	FC	45%	45%	10%	0%	65%	25%	5%	5%	70%	20%	0%	10%
	WP	5%	0%	35%	55%	10%	50%	15%	25%	20%	50%	20%	10%

Table 52

Frequency of Teachers of HNP Group Indicating Topic Emphasis

	1980 Priorities				1981 Priorities				1982 Priorities			
	1	2	3	4	1	2	3	4	1	2	3	4
ND	60%	25%	15%	0%	45%	35%	10%	10%	30%	60%	10%	0%
MG	0%	25%	30%	45%	10%	35%	25%	30%	0%	25%	60%	15%
FC	40%	40%	15%	5%	60%	40%	0%	0%	60%	25%	5%	10%
WP	0%	5%	40%	55%	15%	40%	25%	20%	50%	10%	25%	15%

It appeared that there was little difference in content emphasis among the three groups of students. Typically, number development was given a high priority during the first-grade experiences. There then was a shift to addition and subtraction facts and computations as the highest priority topic area during the second and third years. Measurement and geometry was not given a high priority from any of the teachers during any of the three years of the study. Word problems were given low priority during the first grade, but received increasing emphasis in the two ensuing years.

Teachers' Perceptions of Student Maturity

Teachers were asked to indicate their perceptions of students' arithmetic maturity (AM), intellectual maturity (IM), social/emotional maturity (SEM), and phycho/motor maturity (PMM). They responded to each area on a five point scale, with "1" being immature and "5" mature.

Tables 53, 54, and 55 present data on teachers' perceptions of the students' maturity on the four areas over the three years of the study.

Table 53

Teachers' Perception of Maturity of Students in the LNP Group

		1980					1981					1982				
		Immature				Mature	Im				M	Im				M
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
AREAS	AM	50%	35%	15%	0%	0%	20%	25%	30%	20%	5%	0%	25%	75%	0%	0%
	IM	25%	60%	15%	0%	0%	20%	40%	30%	10%	0%	5%	25%	60%	10%	0%
	SEM	35%	45%	15%	5%	0%	25%	35%	20%	10%	10%	5%	45%	40%	10%	0%
	PMM	20%	35%	40%	5%	0%	15%	20%	40%	15%	10%	10%	25%	50%	10%	5%

Table 54

Teachers' Perceptions of Maturity of Students in the INP Group

		1980					1981					1982				
		Immature				Mature	Im				M	Im				M
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
AREAS	AM	10%	30%	50%	10%	0%	0%	35%	40%	20%	5%	0%	40%	40%	10%	10%
	IM	10%	30%	50%	10%	0%	0%	35%	40%	20%	5%	0%	40%	45%	5%	10%
	SEM	15%	20%	45%	20%	0%	5%	15%	30%	35%	15%	10%	25%	35%	25%	5%
	PMM	10%	35%	25%	30%	0%	0%	20%	40%	25%	15%	5%	20%	45%	20%	10%

Table 55

Teachers' Perceptions of Maturity of Students in HNP Group

		1980					1981					1982				
		Immature				Mature	Im				M	Im				M
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
AREAS	AM	0%	0%	5%	40%	55%	0%	0%	20%	35%	45%	0%	0%	15%	30%	55%
	IM	0%	0%	10%	40%	50%	0%	0%	35%	20%	45%	0%	0%	10%	50%	40%
	SEM	0%	5%	20%	20%	55%	0%	0%	30%	35%	35%	0%	15%	20%	30%	35%
	PMM	0%	0%	15%	30%	55%	0%	0%	20%	45%	35%	0%	0%	35%	25%	40%

The perceptions of three independent groups of teachers on the arithmetic maturity, intellectual maturity, social/emotional maturity and psycho/motor maturity

of the three groups of students in the study are very consistent. The teachers of the HNP group students perceived them as being quite mature on all of the areas during the first-grade experience, and their teachers at the second and third grade continued to perceive them in that way. The modal perceptions of the teachers of the INP group were at the mid-point of the maturity scale, quite consistently each year and on each of the areas surveyed. The LNP group were perceived as being quite immature by their first-grade teachers on all areas of maturity. During the second and third year there was some shift toward perceiving the LNP students as somewhat more mature, but it must be remembered that a majority of them had been held back at the first-grade level and were then being compared with students a year younger. Given this shift in reference group, they were still generally perceived as lacking maturity (very few "4" or "5" rankings) at the end of the third year of the study.

In summary, the programs for the three groups of students entering first grade with marked differences in number abilities were characterized by both similarities and differences. There were differences in the administrative adjustments made by schools; a majority of the LNP group students were retained a grade; only about twenty percent of the INP group students were retained a grade; no HNP group students were retained. There appeared to be some slight differences in program use by the groups. The LNP and INP groups tended to be exposed to more of a variety of programs across the three years of the study; about twenty-five percent of these students in these two groups experienced different programs each year, while only ten percent of the HNP group had such experience. There was also some slight tendency for students in the HNP group to complete a program's grade level work during the year, while there were slightly fewer INP and LNP group students doing so. There were certain years when LNP group students received significantly more supplementary instruction than either of the other

two groups, but other years they received less. The supplementary instruction differences were inconsistent across the years. The nature of the supplementary assistance also differed; the HNP group received enrichment activities; the INP and LNP groups received remedial assistance.

The three groups appeared to receive similar amounts of instruction time in arithmetic daily within the classroom. The pattern of program use was also similar; all groups typically used textbooks or other such program materials daily. The priority of topic emphasis indicated by teachers appeared to be quite similar for all groups across the three years of the study.

There were marked differences in teachers' perceptions of students in the three groups. The HNP students were consistently perceived as being mature in their arithmetic, intellectual, social/emotional, and psycho/motor development. The LNP group, on the other hand, was generally perceived as quite immature in all areas, this in spite of the fact that many had been retained and after the first year of the study were more chronologically advanced than their classroom peers. The INP group tended to be perceived at about the midpoint of the maturity scale by their classroom teachers.

6. Summary and Implications

A cross-section of thirty eight trios of first graders were formed on the basis of number abilities indicated during individual interviews on entering first grade. One member of the trio was assigned to a HNP group on the basis of high performance on rote and rational counting tasks and proficiency in reading and writing numbers. Another member of the trio was assigned to a LNP group on the basis of low performance on rote and rational counting tasks and lack of proficiency in reading and writing numbers. A third member of the trio was assigned to an INP group on the basis of intermediate performance on those tasks relative to the HNP and LNP groups. Each trio member was in the same classroom with the same teacher during the first-grade experience, and each was of the same gender.

On completing the first-grade experience, each member of the trio was followed through the subsequent two years of school experience. Members of the trio were no longer necessarily in intact classrooms, but were in classrooms of choice or district placement. Since the school system was operating under an integration plan, students tended not to be in the same school during the three years of the study.

Of the thirty eight intact trios of students identified on entering first grade, twenty intact trios remained at the end of the third year of the study. When one or more members of a trio was lost, the entire trio was dropped from the study. Evidence was presented that indicated that the remaining twenty intact trios of students were not dissimilar to the thirty eight original trios, and no selectivity factor appeared to be operating.

Twelve addition and subtraction combinations of varying difficulty, four word problems, and a set of tasks designed to elicit student awareness of the commutative and identity properties as they relate to the operations of addition and subtraction were administered to students in individual interview formats. Each student was individually interviewed during the second half of the school year, in

the two years following their first grade experience. Both quantitative and qualitative analyses were carried out.

Classroom teachers responsible for each member of the trio during the three years of the study were asked to describe program characteristics experienced by members of the trio in their classrooms. Teachers were also asked to indicate their perceptions of trio members' maturity each year. The general results were:

1. On the twelve addition and subtraction tasks, product performance (correct answers) of the three groups corresponded to their number ability performance on entry to first grade in both years of the administration. Each group indicated incremental improvement between the first and second years of the administration. The amount of improvement was quite similar for all three groups even though the range of improvement was restricted for the HNP group because of the ceiling on the measurement instrument. In general the HNP, INP and LNP groups retained their relative standing, determined by their number abilities performance on entry to first grade, through their first three years of schooling. However, students in the LNP and INP group above the median in performance on the twelve tasks were performing within the same range of performance as students in the HNP group; those below the median, particularly students in the LNP group, were entirely below the range of performance of the students in the HNP group.

2. An item and error analysis of the twelve tasks indicated some tendency for LNP and INP students to give more inconsistent responses, that is to respond correctly to a task in 1981 and incorrectly in 1982 to the same task. There was also some tendency for LNP students' incorrect responses to be less comprehensible than the incorrect responses made by HNP students. The tendency was for an examiner to be able to understand the reason for many of the errors made by members of the HNP group, while not being able to understand the reason for many errors made by members of the LNP group.

3. Observations by the examiners on the processing procedures used by the students showed marked differences between the groups. The majority of students in the HNP group indicated no overt manifestations during their processing of the twelve tasks other than writing with pencil on the paper. They evidently were applying their knowledge of addition and subtraction combinations in their processing. On the other hand, the majority of students in the LNP group indicated exclusive use of overt manifestations during their processing of the twelve tasks. This usually involved counting on fingers, using helping marks on paper, etc. Although there were differences among the groups on product outcomes (correct answers), these differences were not nearly as marked as differences in the way students in the three groups processed the twelve tasks. These differences correspond to the performance on number abilities on entry into first grade, and existed across both the 1981 and 1982 administrations. There was little change in processing procedures used by students in any of the groups between the 1981 and 1982 administrations.

4. Students were asked to use unit-cubes and ten-rods to demonstrate their responses to two of the tasks, 15 - 8 and 23. Many different strategies were observed. There was quite a marked difference in strategies used by the three groups, especially in the 1982 administration. It appeared that the HNP students were more able to use ten as a whole unit in processing than were students in either of the other two groups; this was especially evident at the time of the 1982 administration. The LNP group tended to use single, discrete cubes in processing demonstrations rather than taking advantage of the efficiency of thinking of ten as a unit whole that could be modeled by a ten-rod. The more mature use of tens and ones materials in the demonstrations corresponded to the performance of the groups on number abilities on entry to first grade by the time of the 1982

$$\begin{array}{r} 23 \\ +14 \end{array}$$

administration. There was less difference, and hence less correspondence, at the time of the 1981 administration.

5. On the four word problem tasks, product performance (correct answers) of the three groups corresponded to their number ability performance on entry to first grade. However, the difference between the LNP group and the ~~LNP~~ group was insignificant at the time of the 1981 administration, but more marked at the time of the 1982 administration. The HNP group performed very well acorss both administrations; there was little room for improvement between the two administrations, but some improvement did take place. The performance of the LNP group was quite poor at the time of the 1981 administration and deteriorated by the time of the 1982 administration. The INP group showed some improvement between the two administrations.

6. Maturity of the strategies used in solving the four word problems corresponded to the number ability performance of the three groups on entry to first grade. The HNP group tneded to use their knowledge of the addition and subtraction combinations in responding correctly. The LNP group tended to use more inappropriate strategies such as simply responding with one of the numbers imbedded in the word problem, and were more often affected by extraneous data in the problem.

7. Students in all three groups appeared to have quite a stable concept of the commutative and identity properties of addition on whole numbers. There was little difference in level of performance among the groups. There were differences among the groups in their evident overgeneralization of commutativity to the operation of subtraction. Most students in the INP and LNP groups apparently believe the order of the numbers in a subtraction situation will not affect the answer. About fifty percent of the HNP performance group indicated that they were aware there would be a difference depending on order.

8. Based on responses from teachers of the students during the three years of the study, the following observations on program were made:

- There appeared to be no difference among the three groups in classroom time spent on arithmetic activities. Typically, it was 30 - 45 minutes daily.
- Few students in any of the groups were in the same arithmetic program all three years of the study. There was a tendency for more students in the INP and LNP groups to experience different programs each year than students in the HNP group.
- There appeared to be little difference in program use among the three groups. Typically, the textbook was used daily. There was a slight indication that more HNP group students completed the text during the year than INP or LNP students.
- Differences in supplementary assistance outside the classroom existed among the groups, but the differences were not consistent. The LNP group received significantly more assistance during the first grade than either of the other two groups, but received less assistance during the second year than either of the other two groups. This evidently came about because of retention in first grade (See #9). One-half of the HNP group received no supplementary instruction during the three years of the study; the percentage was much lower for the INP and LNP groups.
- There appeared to be little difference in the curriculum emphasis given the three groups.

9. It appeared that the most significant adjustment made to accommodate differences in the groups of students was administrative retention. No students from the HNP group were retained a grade; twenty percent of the students in the INP were retained a grade; sixty five percent of the LNP students were retained a grade.

10. Teachers' perceptions of students' arithmetic maturity, intellectual maturity, socio-emotional maturity, and psycho-motor maturity correspond closely with students' number ability on entering first grade across the three years of the study. Teachers of the HNP group saw them as mature in all dimensions across the three years; teachers of the LNP group saw them as immature on all dimensions across the three years. The INP group was perceived as between the two extreme groups in maturity.

A number of implications are suggested from the results of this longitudinal study of school arithmetic development. Some of the results provide evidence that may assist educational practitioners in adapting school arithmetic programming for certain categories of students. Other results may provide researchers some primitive, but useful, insights into the relationship between early number development and subsequent school arithmetic development.

Student performance on simple number tasks on entry to first grade appeared to be quite a valid predictor of school arithmetic performance two and three years later. Students who did not perform well on simple number tasks on entry to first grade appear to be at risk in subsequent school arithmetic development; those who did well were not at risk. So a simple assessment of number skills on entry to first grade appears to give practitioners quite an accurate tool for identifying students who may be at risk in their school arithmetic development.

Performance on number skills on entry into first grade and subsequent school arithmetic development appear to be closely related to more general psychological development. A student with low performance on simple number tasks on entry to first grade will generally be perceived as immature on a number of psychological dimensions two and three years later. Students with high performance on simple

number tasks on entry to first grade will be perceived by teachers as quite mature two and three years later.

Students with different levels of performance on simple number skills on entry to first grade will tend to evidence more marked differences on process procedures than product outcomes when responding to addition/subtraction computation tasks two and three years later. Care should be taken in concluding that if students respond with similar speed and accuracy to a task, they are also processing the task similarly. Students with low performance on simple number skills on entry to first grade appeared to be using less mature process procedures two and three years later than high performance students, even though differences in correct answers to some of the tasks were not great.

Students with low performance on simple number tasks on entry to first grade appear to have special difficulty in developing ability to respond correctly to simple word problems. There appeared to be very little, if any, growth in this ability by those students during their first three years of schooling. Special program consideration may have to be given to these students in the area of word problems.

Most students, regardless of level of number performance on entry to first grade, appear to have quite a stable concept of the identity property and commutative property for the operation of addition. Lower performance students, however, are especially prone to overgeneralizing these properties to subtraction. Special consideration should be given this problem of overgeneralization in arithmetic programming.

Since students with low performance on simple number skills on entry to first grade were still generally immature in their school arithmetic development three years later, programming during these years should be given close scrutiny. It

appeared that there was no dramatic program adaptations made for these students. Formal textbook programs appeared to be the rule no matter what the level of performance on number skills on entry to first grade. If students have quite dramatic differences in number ability on entry to first grade, perhaps dramatic qualitative differences in programming should be made. That would appear not to be happening at the present time.

The administrative procedure of retaining a student for a year did not appear to have an impact on school arithmetic development. Students retained were still perceived as immature by their teachers. Without major qualitative changes in programming, simple retention appeared not to contribute greatly to school arithmetic development for those students who enter first grade with poor numbe- skills.

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